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The Tradeoff Between Return & Risk for Selected Bighorn Basin Crop and Cattle Feeding Systems

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THE TRADE-OFF BETWEEN RETURN AND RISK FOR SELECTED BIG HORN BASIN CROP AND CATTLE FEEDING SYSTEMS

by

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INTRODUCTION

In recent years the prices received and paid by Wyoming agricultural producers have varied widely. Such price changes may be attributed to drought, government policies, variable world-wide stockpiles of crops, fluctuating energy supplies, devaluation of the dollar, expanding and unpredictable fluctuations in foreign trade and other factors which result in changing market conditions for both inputs and outputs. For the producer, such variation contributes to the risk associated with crop and livestock production.

From a producers standpoint, the problem of agricultural decision-making in the face of such risk is significant, given that substantial financial resources are committed to crop and livestock production well in advance of certain knowledge concerning product prices or yield. As a result, major discrepencies may occur between producers' expectations and the income they ultimately realize. When such discrepancies occur, the producer may suffer economic losses, either in the sense of foregone profit opportunities or from failure to have total revenue cover total cost. As the number of cropping or enterprise alternatives available to a producer increases, the more complex the decision problem becomes. Deriving risk-minimizing cropping patterns or systems is often suggested as useful to producers, given that it may provide some measure of the comparative trade-offs between income and risk across alternative cropping systems (Schurle and Erven). As a result, the producer may be made aware of the potential gains and losses associated with a particular system as compared with alternative systems.

While economists recognize the importance of risk in firm level decision making and the subsequent impact of risk on supply response, the definition of risk encountered in the literature is somewhat vague. Knight is generally credited with providing the first precise definition of risk, with risk referring to situations where statistical parameters (or moments such as mean and variance) of the probability distribution can be estimated so as to be actuarially insurable; i.e., a measurable concept. He contrasts risk with uncertainty where uncertainty refers to situations where the parameters cannot be quantitatively determined.

This distinction between these two concepts has become clouded in the more recent literature, to the point where the two terms are on an occasion used almost interchangably. In this study we chose to use the more classical definition of risk; i.e., a measurable concept related to the probability distribution of outcomes. Such a use is motivated in part by the empirical emphasis of the study. That is, risk, as a measure of dispersion such as variance or standard deviation, may be related to the "chance of loss" if the negative variance results in income falling below some actual or minimum level of required producer income. This latter use is in fact consistent with Webster's definition of risk as the "probability of loss" and conforms more closely to producers views of risk. In addition, the analytical framework employed in the subsequent analysis requires a specific measure of dispersion as the risk variable, namely the variance of expected income.

Yahya and Adams, in a recent study of risk associated with crops in Wyoming, examined specific types of risk that the agricultural producer faces in decision making. The most common types relate to prices (both input and output) and yield. For example, producers may have some expectation concerning price for next season's crop, based on observed past prices (the "frequency" with which some range of prices has occurred). However, actual prices may be sharply different, due to changes in the economy over the crop season, or due to purely random phenomena such as the weather. Similarly, actual yields may differ from expected yields due to weather or insect and disease problems. The net effect of this divergence between expected values and those actually realized could be viewed as a potential monetary loss if the actual values are below expectations. Thus, if certain crops have the potential to show large variations between expected values and actual values, these crops may be viewed as more "risky," given that the potential or probability for loss is greater. If the producer has the choice of several crops which may be grown in his production region, he may then elect to reduce his risk by producing crops which have less variability or opt for the potentially higher profits by producing crops with higher risk but higher expected gains.

In the irrigated crop production areas of Wyoming, producers typically have a number of crop alternatives from which to choose. In addition to choosing a crop mix, the respective proportions to be assumed by each crop in that mix must be decided. Given the complexity of the decision problem, there would appear to be a need to expand the above risk concepts into a more comprehensive analysis of the nature and magnitude of risk faced by Wyoming agricultural firms under alternative cropping and livestock systems, as well as to evaluate alternative means for dealing with such risk in decision making. Specifically, a firm level analysis of risk with emphasis on alternative cropping systems would provide useful decision-making information. Although one cannot remove all risk inherent in agricultural production, a producer may choose to limit it to some level via alternative strategies, such as diversification. In such a comprehensive analysis of risk management, it would not be feasible to find the risk-minimizing (or other optimizing) strategy for all producers. Rather, one may attempt to identify the level of risk attendant to such alternatives, with the absolute level of risk assumed by the firm a function of the individual decision-maker's preferences.

Objectives

The overall purpose of this paper is to report some results drawn from a comprehensive risk analysis of crop and cattle alternatives in the Big Horn Basin of Wyoming. These results provide an assessment of the comparative risk associated with various cash crops and cattle feeding systems in the Big Horn Basin. In addition, risk minimizing or risk efficient cropping systems are derived under alternative assumptions concerning income expectations and agronomic or cultural restrictions.

In presenting these results, two sub-objectives are realized. First, the risk attendant to individual crops, cattle feeding alternatives and integrated systems of cash crops and cattle feeding programs are analyzed through the use of budgeting over a 21-year period (from 1956 to 1976). This comparative analysis provides net revenues (gross revenue above variable costs), real values (of the 21 year cash flow), mean net revenues for the 21 year period and for a four-year period (1973-76), standard deviations of the net revenues over the 21-year period, coefficients of variation, and variance-covariance matrices for the various crops, cattle feeding alternatives and integrated systems.

The second objective will be to estimate expected net income-variance (E,V) frontiers for selected crops and integrated systems in the Big Horn Basin irrigated crop area. Such frontiers provide an explicit measure of the relationship or trade-off between income derived from a cropping system and variability (variance) of that system. The E,V frontiers utilize a range of different crops that are grown in the Basin, plus the various cattle feeding programs which utilize internally generated feed crops. The resulting frontiers may be viewed as providing an efficient set of alternative cropping and feeding systems (i.e. crop and livestock systems with a minimum level of variance for a given level of expected income).

Area of Study

The empirical thrust of this paper is irrigated crop production within the Big Horn Basin. This area includes Park, Big Horn, Washakie and Hot Springs counties. The Basin is delineated by the Big Horn Mountains on the east, the Absaroka Range of the west and the Owl Creek Mountains which form the southern boundary. The Big Horn River flows north through the Basin and has two major tributaries, the Greybull and the Shoshone Rivers. Along the floodplains of the Big Horn, Greybull and Shoshone Rivers most of the 212,000 acres of irrigated crop production occurs. The major crops produced in the area are sugar beets, malting barley, feed barley, alfalfa, corn for grain, corn for silage and dry beans. (Wyoming Agricultual Statistics) Such a mix of crops can be produced in this area given a growing season which varies from 120 frost-free days at Cody to 133 frost-free days at Worland.

Nationally, Wyoming ranks 11th in dry beans, 10th in barley, 9th in sugar beets, 24th in alfalfa, 25th in cattle on feed, 29th in corn for silage and 37th in corn for grain. In terms of state production, 65 percent of the state's sugar beets, 44 percent of the barley, 38 percent of the dry beans, 18 percent of the corn and 23 percent of the total alfalfa are produced in the Big Horn Basin (Wyoming Agricultural Statistics, 1978).

For the 1977 crop year, approximately 42 percent of the 212,000 irrigated acres were in alfalfa, 31 percent in barley, 15 percent in sugar beets, 6 percent in corn for silage, 4 percent in dry beans, 2 percent in corn for grain and 1 percent in minor crops (wheat and oats). The 212,000 acres excludes a significant amount of native hay production, used primarily to maintain livestock breeding herds. A percentage of the alfalfa raised in the Big Horn Basin is also used as winter feed for maintainence of breeding herds of cattle and sheep. This study will deal only with irrigated crop production, analyzing the returns and respective risks associated with alternative cropping systems.

The Big Horn Basin was selected as an appropriate study area based on several criteria. First, the physical environment makes it conducive to economic study. That is, it is a closed basin with similar weather patterns and displays relatively homogeneous soil and water conditions. Second, it is one of the major cash crop areas of Wyoming, producing a major portion of the state's sugar beets, barley and dry beans (noted above). In addition to a relatively diverse set of crops, 15 percent of the state's cattle and 13 percent of the sheep are found within the Basin (Wyoming Agricultural Statistics, 1978).

The following section (Section II) provides a brief overview of the procedures and assumptions used in the analysis as well as simple statistics (mean, standard deviations and coefficient of variation) for the major crops and cropping systems of the Big Horn Basin. The discussion of risk attendant to agricultural production is largely qualitative in nature. Section III extends these results into a more explicit assessment of the trade-offs between income and risk through the derivation of E,V frontiers. Alternative specification of the income and risk parameters are reported. These latter results provide information on a wide range of potential crop and cattle systems.

SECTION II

COMPARATIVE ANALYSIS OF ALTERNATIVE CROPS AND CATTLE FEEDING SYSTEMS: PROCEDURES AND RESULTS

This section provides a brief explanation of the procedures employed in the analysis, as well as a discussion of the comparative incomes and risk for the crops and systems. The procedural discussion will also cover an important component of the study, the initial interview of individual producers in the Big Horn Basin. The data obtained from these interviews, when combined with secondary data, are used to derive net revenues (above variable costs) for each crop and cattle alternative. The data set compiled here is then used to provide a measure of the risk associated with agriculture in this area. Finally, the information generated in this phase of the study is instrumental in the assessment of the trade-off between risk and income associated with alternative agricultural systems, i.e. the E,V frontier estimation as discussed in Section III.

Specifically, the first part of this section examines the method used in the generation of primary data (interviews) and results of this data generation. The procedures used to estimate net revenues for the different crops, cattle alternatives and integrated crop and cattle systems are then defined. The variance-covariance matrices, correlation matrices, means, standard deviations and coefficients of variation (variability measures) associated with this set of net revenues were also computed (the resultant matrices are presented in Woolery). These generally descriptive results may be viewed as providing a first approximation to the inherent risk that producers face. Such information may then be used by producers to develop efficient farm plans based on producer's individual preferences with respect to income and risk. In addition, these results also provide the input data needed to estimate the E,V frontiers for a more complete assessment of risk in farm planning.

Generation of Primary Data--Producer Interviews

Eight producers were interviewed, ranging in age from young individuals just starting in agriculture to older, well-established producers, with a range in size of operation from under 200 acres to over 1,000 acres. The mean size of farm in the interview was about 575 acres. This mean size farm, plus other crop and input data provided by the producers, formed the empirical basis for much of the study analysis. Specifically, data were collected concerning the mix of crops (acreage planted to each), extent of involvement in cattle feeding, past cropping and feeding histories, and their personal views on risk and the nature of farming. In addition, the conversations with producers suggested areas of perceived informational need.

In terms of inherent risk, producers generally indicated that they viewed agriculture as a "risky business". In describing their view of risk, most provided definitions closely approximating the more formal statistical views of risk. That is, in general they viewed risk as the deviation of income from some normal or expected level. Also, they indicated that some crops have greater risk than others, e.g. most viewed dry beans as a very risky crop. At the same time, those interviewed realized that to raise the lower-risk crops exclusively would result in correspondingly low incomes. For this reason, most of those interviewed raised a minimum of alfalfa, primarily to facilitate crop rotation (to improve soil structure). For those producers involved in cattle feeding, rotational considerations also affected crop mix, with their primary feed being corn silage and/or feed barley, in addition to some alfalfa hay. Those producers using cash cropping systems tended to use a cropping system made up of equal proportions of malting barley and sugar beets with little or no alfalfa rotation. Very few producers raised dry beans. Of those that did, only one viewed it as his major crop in terms of acreage.

Based on the information gathered in this first set of interviews, estimated net revenues for the alternative cropping and feeding programs in the Big Horn Basin were calculated. The procedure used in this estimation is discussed in the following section.

Estimation of Net Revenues for Selected Agricultural Enterprises

Four sets of net revenues were estimated in the study. In the first set, net revenues were computed for each crop. Then, net revenues were calculated for six cattle feeding alternatives. The third set consists of net revenues for combinations of feed crops and cash crops to simulate the total net income to the hypothetical 575 acre farm if feed crops were sold in place of feeding cattle. Finally, the net revenues for combined cash crop and cattle feeding alternatives were computed to provide total farm net income under such an integrated system.

These net revenues were then used to compute such simple statistics as real values (over a 21-year period), means, standard deviations, coefficients of variation, variance-covariance matrices and correlation matrices for each of the above four sets of net revenues. These measures provided the basis for a descriptive analysis of Big Horn Basin cropping practices, as well as providing data for the E,V analysis which follows (i.e. the net revenues and variance-covariance matrices needed to estimate the E,V frontiers).

Estimation of Net Revenues for Individual Crops

The crops included in the analysis are: alfalfa hay, grain corn, silage corn, dry beans, malting barley, feed barley and sugar beets. The value of the net revenues were based on data obtained from Yahya and Adams for the period 1956-74, where net revenue is defined as being equal to gross revenue less the variable cost of production. The data in the current study were extended through the years 1975 and 1976. The analysis thus covered a period of 21 years, 1956-76. All calculations were performed on a dollar per acre basis. Also, for each of the crops a real value was calculated over the period of analysis using an index of prices received by farmers (See Table 1, footnote b) to remove the effect of inflation on net revenues.

Estimation of Net Revenues for Cattle Feeding Alternatives

Producers indicated an interest in a comparison of income to be obtained from feeding cattle with that obtained from producing cash crops. To facilitate this analysis, the hypothetical farm acreage was initially proportioned between 345 acres allocated to feed crops and 230 acres for cash crops (alternative proportions of feed crops and cash crops were also evaluated, as discussed in Section III). The feed crop alternatives were: corn for silage, alfalfa hay and feed barley. Also, sugar beet tops were assumed to be cut, windrowed and left in the field for consumption. Two cash crop alternatives were included, (for the 230 acres allocated to cash crops) one in which 167 acres of sugar beets and 63 acres of malting barley were produced, and another in which the entire 230 acres were planted to sugar beets.

Three possible cropping systems were developed on this hypothetical farm: (1) crop alternative 1 had 167 acres of sugar beets, 63 acres of malting barley, 161 acres of corn for silage, 75 acres of alfalfa hay and 109 acres of feed barley. This breakdown of acreage, as well as the overall acreage of 575 acres, was based on information obtained via the interviews; (2) crop alternative 2 featured 230 acres of sugar beets, 161 acres of corn for silage, 75 acres of alfalfa hay and 109 acres of feed barley. Thus in this system all of the cash crop acreage was used for sugar beets with the feed acreage remaining the same as the above; and (3) crop alternative 3 included 167 acres of sugar beets, 63 acres of malting barley, 235 acres of corn for silage and 110 acres of alfalfa hay. In this alternative the cash crop breakdown remained the same as in alternative 1, with no feed barley being raised, increasing the acreage in corn for silage and alfalfa hay. A summary of these alternatives is presented in Table 1.

Table 1. Specific Crop Acreages for Three Alternative Cropping Systems on the 575 Acre Hypothetical Farm.

Crop	Cropping Systems Alternative 1	 Alternative 2 (acreages)	 Alternative 3
Cash Crops			
Sugar beets	167	230	167
Malting barley	63	0	63
Total Cash Crops	230	230	230
Feed Crops			
Corn silage	161	161	235
Alfalfa hay	75	75	110
Feed barley	109	109	0
Total Feed	345	345	345
Total Crop Acreage	575	575	575

Under each of the three crop alternatives, four alternate feeding programs were initially developed as presented in Woolery:

(1) Buy 400 lb. steer calves and feed them at a projected rate of gain of 1.5 lbs. per day, then sell them when they reach 675 lbs. as yearlings.

(2) Buy 500 lb. steer calves and feed them at a rate of 2.0 lbs. per day gain, then sell them when they reach 700 lbs. as yearlings.

(3) Buy 500 lb. steer calves and feed them at the rate of 1.5 lbs. per day gain, selling them when they reach 775 lbs. as yearlings.

(4) Buy 500 lb. steer calves and feed them at the rate of 2.0 lbs. per day gain, selling them when they reach 800 lbs. as yearlings.

Combining the above crop and feeding assumptions resulted in 12 alternate feeding programs (three crop alternatives by four feeding regimes). However, given the general superiority of the feeding at higher rate of gain (i.e. 2 pounds per day), and the consistent relationships observed within this weight gain class as compared to the 1.5 pound weight gain, only the higher rate of gain will be presented and assessed in this report.[1/] This results in six cattle alternatives to be formally discussed.

All calves were assumed to be purchased on November 1 and sold by April 30 of the following spring (181 days). Using the feed provided by these crop mixes the number of steer calves to be fed were computed based on National Research Council requirements.[2/] In computing the net revenues for the cattle alternatives the following costs were included: purchase cost of the calves, non-feed costs (repairs to corrals, veterinary costs, etc.), feed preparation costs, cost of production of the feed, marketing costs and interest costs. In addition, a death loss was included in the calculation of the gross revenue. The costs and other data used in these calculations are drawn from Agee.

Each of these six feeding alternatives were calculated over 21 years (the period 1956-76). A variance-covariance matrix, correlation matrix, mean, standard deviation, coefficient of variation and real value were also computed for each of the feeding alternatives and two cash cropping alternatives (separately), using the net income figures calculated above.

Estimation of Net Revenues for Integrated Feed-Cash Crop Systems

Feed producers within the Big Horn Basin may choose either to feed their crops to cattle or market the feed directly. The net revenues associated with direct sales of the feed crops were calculated as a basis

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[1/] The complete analysis of the 12 alternatives is presented in Woolery.

[2/] The number of calves that could be fed under each alternative were calculated following the procedure discussed in Woolery. The interested reader is referred to that publication for a more detailed discussion of procedures.

for comparison with the income to be obtained by the feeding and sale of cattle. Net revenues for the feed crops were calculated by multiplying the net revenues for individual crops by the percentage of each crop in the feed mix associated with that cattle alternative, to arrive at an income measure for the 575 acres. In the analysis of the cattle feeding alternatives, the feed crop programs provided the major source of feed. For the analysis of the direct sale of feed, these feed crop programs were designated A and B. Feed crop mix A had 46.7 percent corn for silage, 21.7 percent alfalfa hay and 31.6 percent feed barley of the total of 345 acres (or 60 percent of total acreage). This is the feed mix used to calculate cattle feeding alternatives 1, 2, 3 and 4. Feed crop mix B featured 68.3 percent of the feed crop acreage devoted to corn for silage, with the remaining 31.7 percent in alfalfa hay. Feed mix B was used to develop cattle feeding alternatives 5 and 6. In addition to these feeds, some sugar beet tops were provided as discussed in the preceeding section.

The two feed crop alternatives calculated above were combined with the cash crop programs employed in the cattle feeding programs to yield the total net revenues provided to the farm (575 acres) if the feed crops were sold in place of the cattle. The cash crop programs represent the remaining 230 acres (or 40 percent of total acreage) on the hypothetical unit. They are designated I and II. Cash crop program I includes 167 acres of sugar beets and 63 acres of malting barley on these 230 acres, whereas program II devoted the entire 230 acres to sugar beets. A summary of the respective feed and cash crop systems utilizing the 345 acres devoted to feed crops and 230 acres devoted to cash crops are presented in Table 2. The net revenues for each of the two cash crop programs were obtained by multiplying the respective crop mix percentage by the net revenues of individual crops developed above.

Table 2. Individual Feed and Cash Crops Programs, by Crop Acreage and Percentage.

Crops	Feed Crops	
	Mix A	Mix B
Corn silage	161 (46.7%)	235 (68.3%)
Alfalfa hay	75 (21.7%)	110 (31.7%)
Feed barley	109 (31.6%)	0 (0%)
Total	345 (100.0%)	345 (100.0%)
	Cash Crops	
Sugar beets	167 (72.7%)	230 (100.0%)
Malt barley	63 (27.3%)	0 (0%)
Total	230 (100.0%)	230 (100.0%)

Finally, the two feed crop alternatives and the two cash crop alternatives were combined to yield three distinct cropping systems, as presented in Table 3. Crop alternative 1 (IA) had 29 percent sugar beets, 11 percent malting barley, 28 percent corn for silage, 13 percent alfalfa hay and 19 percent feed barley. Crop alternative 2 (IIA) featured 40 percent sugar beets, 28 percent corn for silage, 13 percent alfalfa hay and 19 percent feed barley. Crop alternative 3 (IB) included 29 percent sugar beets, 11 percent malting barley, 41 percent corn for silage and 19 percent alfalfa hay. These three crop alternatives approximate the range of systems suggested by the interviews.

Table 3. Integrated Feed and Cash Crop Systems for 575 Acre Farm, Crop Acreage and Percentage.

	Cropping System		
Crop	IA	IIA	IB
Sugar beets	167 (29%)	230 (40%)	167 (29%)
Malt barley	63 (11%)	0 (0%)	63 (11%)
Corn silage	161 (28%)	161 (28%)	235 (41%)
Alfalfa hay	75 (13%)	75 (13%)	110 (19%)
Feed barley	109 (19%)	109 (19%)	0 (0%)
Total	575 (100%)	575 (100%)	575 (100%)

The above net revenues were all computed on a dollar per acre basis. Means, standard deviations, coefficients of variation and real values were calculated for the seven programs above, i.e. feed crop programs A and B, cash crop programs I and II and the combined crop programs IA, IIA and IB. Variance-covariance matrices and correlation matrices were computed for the two feed crop programs and the three combined crop programs. The two cash crop programs were included in the variance-covariance matrix and correlation matrix calculated for the cattle feeding programs above. This was done to find the covariance terms between the cattle feeding alternatives and the cash crop programs.

Estimation of Farm Net Revenues
for Integrated Cattle-Cash Crop Systems

In order to assess the effects of cattle feeding on farm income, it was necessary to combine the cattle feeding programs with the cash crop programs to provide total farm income under the six cattle feeding alternatives. Revenues were estimated for the integrated cattle-cash crop systems to provide an estimate of the total net income resulting from a land utilization mix of 40 percent cash crops and 60 percent cattle feeding (the same 60 percent allocated to feed crops above). These net revenues were obtained by multiplying the net revenue per acre for each alternative plus the net revenue per acre from the individual crop section, times the number of acres planted to each cash crop for each cropping alternative. These results were calculated over the same 21-year period (1956-1976). Variance-covariance matrices, correlation matrices, means, standard deviations, coefficients of variation and real values were calculated for each of the 12 systems using the total net revenue obtained for each.

In summary, net revenues were calculated for:

(1) A set of crops commonly grown within the Big Horn Basin, including both feed and cash crops;

(2) Six distinct cattle feeding alternatives with each alternative based upon assumptions concerning the allocation of crop production between specific cash and feed crops and purchase weight and rate of gain of cattle;

(3) A set of alternatives integrating feed crops with cash crops for the 575 acre hypothetical farm; and

(4) A set of alternatives integrating cattle feeding systems with cash crops for the 575 acre farm.

A Comparative Assessment of Incomes and Variability

Using the alternative crop and cattle feeding systems outlined above, comparisons can be made with respect to incomes and other relevant characteristics of the systems. The following results include the incomes and attendant statistical measures for the crop and cattle alternatives over the 21 year period of analysis.

Net Revenues, Real Values and Variability Measures for Individual Crops

The results of the first part of the comparative analysis dealing with each crop are presented in Table 4. The means were initially calculated from the entire series of data (1956-1976). In addition, means were calculated based on the last four years of data for each crop. It was felt that net revenues from this later period would more closely represent the income expected currently, and may be closer to producers' expectations concerning present and future prices. This shorter time period may be viewed as a measure of "subjective" mean net revenues in the sense that they more closely approximate the subjective views of producers. The standard deviations of net incomes were calculated for the entire time series. Based upon the above means and standard deviations, coefficients of variation were calculated for each of the two means ("objective" or the entire 21 year period and the "subjective"), where the coefficient of variation is found by dividing the standard deviation by either the objective or subjective mean. Comparison across crops cannot be made between the two respective coefficients of variation; comparisons of variability across crops must be within the same time series.

The coefficient of variation may be viewed as a measure of the relative "riskiness" of individual crops. In a statistical sense it measures the relative width of the distribution of net revenues around the mean. Thus, a high coefficient of variation is generally considered more risky relative to a lower one. In the case of the subjective (1973-1976) coefficient of variation, it is not a pure statistical measure in that it contains a mean from one data set and a standard deviation from another. It can, however, be used to compare crop or feeding alternatives within

Table 4. Net Revenues, Coefficients of Variation and Real Values for Big Horn Basin Crops.[a/]

Crop	Mean 1956-76	Mean 1973-76	Standard Deviation 1956-76	Coefficient of Variation 1956-76	Coefficient of Variation 1973-76	Real Value[b/]
Alfalfa hay	$ 24.64	$ 67.05	$ 22.28	.9042	.3323	$ 380.50
Corn for grain	38.13	105.85	37.61	.9864	.3553	573.00
Corn for silage	35.94	95.03	35.22	.9800	.3706	543.38
Dry Beans	61.16	211.02	95.56	1.5625	.4528	854.67
Malting barley	178.60	363.34	101.94	.5708	.2806	2930.39
Feed barley	43.81	112.02	35.74	.8159	.3191	683.12
Sugar beets	129.18	384.04	159.85	1.2374	.4162	1914.77

[a/] Net revenue (or gross margin) is obtained by subtracting variable costs from the annual gross revenue for each crop; it represents a return to land and management. All values on a dollar per acre basis.

[b/] "Real Value" as used here represents net revenue over the 21 year period as measured in real dollars; i.e., deflated by the USDA index of prices received by farmers (Agricultural Statistics).

Data sources: 1) Yahya and Adams. "Some Measures of Price, Yield and Revenue Variability for Wyoming Crops and Cropping Systems," RJ 115, September 1977.
2) Agee. Various crop studies for the Big Horn Basin.
3) Wyoming Agricultural Statistics, 1978.

the relative set of subjective mean results throughout the study.

Another phase of the descriptive or qualitative comparison of crops and systems was the calculation of real values for each of the crops. This was done by taking the net revenues for each year for each crop and dividing them by an index of prices received to remove the effect of inflation. The time series covered the period from 1956-1976, with all values adjusted to 1967 dollar equivalent.

The results presented in Table 4 are not too surprising when viewed in the light of the general opinions of producers in the area. As expected, malting barley and sugar beets showed the highest mean net revenues and real values, with feed crops falling in the lower range of mean net revenues and real values. Again as expected, dry beans showed the highest coefficient of variation among the individual crops. The other coefficients of variation followed their mean net revenues, i.e. high mean, high coefficient of variation (with exception of malting barley which had a high mean net revenue and real value with a relatively low coefficient of variation). This may explain in part the current popularity of this crop in the area.

Net Revenues, Real Values and Variability Measures for Cattle Feeding Alternatives

An alternative to the use of the 575 acre land base for production of cash crops is to feed cattle with forage produced on the farm. This section compares the relative incomes and variations in those incomes for selected cattle regimes. Major point of comparison in the analysis include: (1) the comparison of profitability across two starting weight classes, 400 lbs. vs. 500 lbs.; (2) differences in the coefficients of variation and real values between cattle and the individual crops presented in Table 1; and (3) changes in coefficients of variation as cattle are integrated with the individual crops into a complete farm system. These results are presented in Table 5.

The results of this analysis show a much higher risk (i.e. coefficient of variation) associated with the heavier starting weights of steer calves, without a compensation in higher net revenues. This pattern holds for each of the three heavier weight cattle alternatives (500 lb. starting weight). The 400 lb. weights have higher real values and lower coefficients of variation for the period 1956-76 than do the feed crops (alfalfa, corn for silage, corn for grain and feed barley) and 500 lb. cattle alternatives. If one examines the changes in the income means for cattle between the 1956-1976 period and the 1973-1976 period, the increase in income is small compared to the mean net revenue changes observed for the individual crops. This is due largely to the agricultural price situation observed during the 1973-1976 time period. Crops experienced favorable movements in prices, while cattle were in a period of downward price adjustments.

To complete the comparative discussion of crops versus cattle feeding, one final alternative should be introduced into the analysis i.e. the direct sale of feed crops in place of cattle. These results are presented below.

Net Revenues, Real Values and Variability Measures for Integrated Feed-Cash Crop Systems

An assessment of the returns to be realized from the sale of the mix of crops used in calculating the feeding programs for the cattle (i.e. direct sale of feed rather than feeding cattle) are presented in Table 6.

Table 5. Net Revenues, Coefficients of Variation and Real Values for Cattle Feeding Alternatives.[a]

Programs[b]	Mean 1956-76	Mean 1973-76	Standard Deviation 1956-76	Coefficient of Variation 1956-76	Coefficient of Variation 1973-76	Real Value[c]
1	$75.31	$88.67	$58.85	.7814	.6637	$1354.74
2	51.57	58.47	56.42	1.0939	.9649	918.03
3	78.44	92.88	60.41	.7701	.6504	1410.93
4	53.90	61.87	58.01	1.0763	.9376	958.99
5	87.10	98.12	66.94	.7685	.6822	1574.52
6	59.30	66.28	63.98	1.0789	.9653	1056.80

[a] All values on a dollar per acre basis.

[b] In all feeding programs, steer calves are purchased November 1, at different purchase weights, fed different rates of gain on different crop combinations, and sold by April 30. The specific combinations are:

1. 400 lb. calves, 2.0 lb/day gain on corn silage, feed barley, alfalfa hay, beet tops pasture.
2. 500 lb. calves, 2.0 lb/day gain on corn silage, feed barley, alfalfa hay, beet tops pasture.
3. 400 lb. calves, 2.0 lb/day gain on corn silage, feed barley, alfalfa hay, beet tops pasture.
4. 500 lb. calves, 2.0 lb/day gain on corn silage, feed barley, alfalfa hay, beet tops pasture.
5. 400 lb. calves, 2.0 lb/day gain on corn silage, alfalfa hay, beet tops pasture.
6. 500 lb. calves, 2.0 lb/day gain on corn silage, alfalfa hay, beet tops pasture.

[c] See Table 1, footnote b.

Table 6. Net Revenues, Coefficients of Variation and Real Values for Integrated Feed-Cash Crop Systems.[a]

	Mean 1956-76	Mean 1973-76	Standard Deviation 1956-76	Coefficient of Variation 1956-76	Coefficient of Variation 1973-76	Real Value[c]
Feed Crop Programs[b]						
A	$ 35.97	$ 94.32	$ 31.08	.8639	.3295	$ 567.10
B	32.34	86.11	30.30	.9369	.3519	505.68
Cash Crop Programs						
I	142.72	378.40	139.01	.9746	.3676	2212.79
II	129.18	384.04	159.85	1.2374	.4162	1961.53
Cash Crops and Feed Programs Combined						
IA	78.67	207.95	70.94	.9017	.3411	1242.33
IIA	73.26	210.21	78.21	1.0676	.3720	1124.85
IB	76.49	203.03	69.31	.9062	.3414	1205.53

[a] All values on a dollar per acre basis.

[b] The underlying crop mix in each program is as follows:
Feed Crop Program A: Corn silage (46.7%), alfalfa hay (21.7%), feed barley (31.6%); 60% of total acreage.
Feed Crop Program B: Corn silage (68.3%), alfalfa hay (31.7%): 60% of total acreage.
Cash Crop Program I: Sugar beets (72.6%), malting barley (27.4%); 40% of total acreage.
Cash Crop Program II: All sugar beets; 40% of total acreage.
Cash Crop & Feed Crop Program IA: Sugar beets (29%), malting barley (11%), corn silage (28%), alfalfa hay (13%), feed barley (19%).
Cash Crop & Feed Crop Program IIA: Sugar beets (40%), corn silage (28%), alfalfa hay (13%), feed barley (19%).
Cash Crop & Feed Crop Program IB: Sugar beets (29%), malting barley (11%), corn silage (41%), alfalfa hay (19%).

[c] See Table 1, footnote b.

Perhaps the most significant observation is that the feed crop programs (A and B) alone show a much lower 1956-1976 mean and real value than the corresponding cattle alternative, with about the same coefficient of variation. The feed crop 1973-1976 means are about the same as the cattle alternatives for the 400 lb. weight classes, but show a much lower 1973-1976 coefficients of variation than the cattle. Again, this reflects the relative price relationships between cattle and feed observed in the latter period.

The cash crop programs I and II in isolation have results which are expected, given that they represent a mix of sugar beets and malting barley (I) or all sugar beets (II). Both have high net revenues and present values, with corresponding levels for coefficients of variation.

Finally, the cash crops and feed crops programs were combined to simulate the effect of selling the entire crop mix in place of feeding part to the cattle. One point to note is that the combined crop mixes have higher real values and means than for the feed crops programs, but they are lower than cash crop programs. The coefficients of variation for the integrated feed-cash crop programs lie between the cash crop programs and the feed crop programs.

Net Revenues, Real Values and Variability Measures for Integrated Cattle-Cash Crop Systems

The final step in the analysis of alternative systems is to combine the cash crops and cattle feeding programs to provide total farm income. The results of this analysis are presented in Table 7. The major points to be gleaned from the table are: (1) the 1956-1976 mean net revenues for the combined cash crop and cattle feeding programs are higher than for

Table 7. Net Revenues, Coefficients of Variation and Real Values for Integrated Cattle-Cash Crop Systems.[a]

Programs[b]			Mean 1956-76	Mean 1973-76	Standard Deviation 1956-76	Coefficient of Variation 1956-76	Coefficient of Variation 1973-76	Real Value[c]
I	A	1	$102.21	$204.56	$69.70	.6819	.3407	$1713.56
I	A	2	88.03	186.44	67.79	.7700	.3636	1452.95
II	A	3	98.74	209.34	75.49	.7646	.3606	1631.19
II	A	4	84.01	190.74	73.64	.8765	.3860	1359.99
I	B	5	109.35	210.23	71.20	.6511	.3386	1836.35
I	B	6	92.67	191.13	70.22	.7577	.3674	1536.12

[a] All values on a dollar per acre basis.

[b] I and II are the cash crops programs. A and B are the feed crop programs (i.e., not sold but fed to cattle). 1-6 refer to the cattle feeding programs (see Table 5).

[c] Same as Table 4.

the cash crops combined with the feed crops, as presented in Table 6; (2) the 1956-1976 coefficients of variation are lower for the combined cattle and cash crops than for the feed and cash crops programs (Table 6); and (3) the 1973-1976 coefficients of variation and means are about the same for both cattle-cash crops and feed-cash crops alternatives (Table 6). The most significant observation is that the real values of cattle-cash crops programs are higher than for the cash-feed crops programs. This, combined with the lower 1973-1976 coefficients of variation, indicates the superiority of combining cattle feeding with cash crops to reduce risk, as measured by the coefficient of variation.

This discussion has covered relevant descriptive and statistical information concerning seven individual crops and six cattle feeding alternatives for the Big Horn Basin. The alternatives have also been integrated to simulate the total or aggregate risk associated with selected combinations. These systems, under varying assumptions with respect to income and variance are analyzed within a total farm, E,V framework to arrive at "optimal" or risk minimizing crop combinations, as presented in Section III.[3/]

A Note on the Use of Detrended Data in the Generation of Risk Measures

The standard deviations, variance-covariance matrices and correlation matrices discussed above were estimated using actual or nominal time series data on net revenue as obtained from the cited sources.

[3/] The net revenue data (income) and variance-covariance matrices estimated for each of the cases above and as used in the subsequent E,V analysis are presented in Woolery.

Any trend effects contained in the time series data were not removed. The variance measures may thus be viewed as representing total variation due to all causes. An alternative approach is to "detrend" the data to arrive at the random variation or variance. Detrended data of this type, and the resulting variances, are frequently employed in the calculation of risk measures such as variability coefficients[4/] as well as E,V frontiers (see, for example, Halter and Dean or Lin et al). These researchers have argued that this "random" variability measured from detrended data is a truer measure of risk than that obtained via use of nominal or actual data. The following section presents results based upon use of such detrended data.

Results of Detrended Data

Table 8 presents a summary of detrended coefficients of variation[5/] and the random standard deviations. The removal of the trend from the data resulted in a decrease in coefficients of variation for individual crops between 37 percent and 73 percent, with alfalfa decreasing 73 percent, corn for grain and silage about 54 percent, malting barley 60 percent, dry beans 38 percent and sugar beets 37 percent. However, for the cattle the coefficients decreased by only 13-14 percent, far less than for the crops. These results would indicate that of the total

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[4/] Variability coefficients differ from the coefficients of variation in that the standard deviation in the former is calculated from detrended data, thus representing a "random" standard deviation. The procedure used to detrend the data series is discussed in Yahya and Adams.

[5/] As noted in footnote 4 above, use of the random measures of variance are typically referred to as a "variability coefficient". However, for ease of presentation the term "coefficient of variation" will be used, in both cases; i.e., random and total variance.

Table 8. Summary Table of Random Data (Detrended) Results for Six Cattle Feeding Alternatives and Seven Crops.[a]

	Mean 1956-76	Mean 1973-76	Standard Deviation 1956-76	Coefficient of Variation 1956-76	Coefficient of Variation 1973-76
Cattle Feeding Alternatives					
1	$ 75.31	$ 88.67	$ 50.77	.674	.573
2	51.57	58.47	56.42	1.09	.925
3	78.44	92.88	52.07	.664	.561
4	53.90	61.87	58.01	1.08	.912
5	87.10	98.12	58.04	.666	.592
6	59.30	66.28	63.99	1.08	.960
Cash Crops					
Alfalfa	24.64	67.05	6.00	.243	.089
Corn for grain	38.13	105.85	17.37	.455	.164
Corn for silage	35.94	95.03	15.97	.444	.168
Dry beans	61.16	211.02	59.13	.967	.280
Malting barley	178.60	363.34	41.04	.230	.113
Feed barley	43.81	112.02	10.96	.250	.098
Sugar beets	129.18	384.04	101.35	.785	.264

[a] All sales on a dollar per acre basis.

variability observed in the cattle feeding alternatives, most of the variability may be attributed to the random component.

The major change observed in the variance-covariance and correlation matrices after detrending was an increase in negative correlation (See Woolery, Appendix A). All crops, with the exception of malting barley, displayed negative correlation with the six cattle alternatives. This is an important observation in that negative correlations between crops imply that such crops will work well in a diversification scheme, i.e. tend to have more stable incomes. Also as expected, the variance and covariance terms decreased in size with the removal of the trend from the initial data.

Summary

The comparative analysis of alternative crop systems indicates that feed crops (alfalfa, corn for grain, corn for silage and feed barley) have lower risk but correspondingly lower returns than the cash crops. Dry beans showed the highest degree of risk, but with a lower income than either malting barley or sugar beets.

In the case of the cattle feeding alternative, it was observed that the lower starting weights for cattle improved the risk-to-return ratio. Also, the cattle alternatives (using objective means 1956-1976) were superior to all crops except malting barley and sugar beets. However, using the more subjective income (mean for period 1973 to 1976) cattle were more risky than crops. Similar results were indicated for the combined crop programs. However, in the case of the combined cash crops and cattle alternatives, the cattle were superior or equal to the crop programs. The major implication is that the combining of cattle with cash crops tends to reduce the overall risk.

The process of detrending the data to arrive at alternative variance estimates resulted in a decrease in variance terms, as expected. This has the effect of lowering the coefficient of variation for both the cattle and crop alternatives. However, the decrease in variance was much greater for crops than for the cattle alternatives, which indicates that the random component of total variance was greater for cattle than crops. In addition, the number of negative correlations between specific crops and cattle feeding systems increased, implying risk minimizing practices to be discussed in the following E,V analysis.

SECTION III

E,V FRONTIER ANALYSIS

The preceeding discussion has dealt with comparative statistics concerning individual crops or selected systems where crop mixes are in fixed proportions. The resultant values on incomes, standard deviations and coefficients of variation can be used to assess the relative "riskiness" of the individual crops or systems via a rank ordering of coefficients of variation. The information on incomes also provides some absolute measure of the pay-off for assuming a given level of risk.

This information while useful does not explicitly address the trade-off between risk and income that occurs as a producer alters his cropping mix or adds cattle feeding to an existing crop mix. It is generally assumed that as more enterprises are added to a farm plan the lower will be the variability of incomes. This is the rationale for diversification as a risk-minimizing strategy. The case for diversification arises from the number of negative income correlations that exist between potential crops within that system (i.e. if two crops display negative correlations in their incomes they represent a "good" diversification combination). However, among a given set of agricultural enterprises, the number of potential crop combinations and acreage proportions of that mix are extremely large. One means of articulating the large number of potential combinations and their resultant incomes and risk (variance) is through a mapping of incomes and variance for different crop combinations. The resulting curve or frontier then graphically captures the trade-off between incomes and risk for an almost infinite number of combinations. The E,V frontier, as developed by Markowitz, is one means of arriving at such an explicit trade-off relationship.

The E,V Framework

The E,V framework may be viewed as providing an efficiency frontier of the set of possible farm plans, where efficiency is measured in terms of the relationship between expected income and corresponding variance. Those points (or plans) that lie on the E,V frontier are superior to those which lie below and to the right of the curve, given that the E,V curve presented here is concave to the origin. In this respect any point that does not lie on the frontier is dominated by a plan that lies to the left, which has the same level of variance but with a higher level of income.

In an empirical context, the E,V framework may be characterized by the following definitions:

$$\text{Expected Net Income:} \quad E = \sum_{i=1}^{n} q_i e_i \qquad (1)$$

$$\text{Variance of Net Income:} \quad V = \sigma^2 = \sum_{i=1}^{n} \sum_{i=1}^{n} \sigma_{ij} q_i q_j \qquad (2)$$

where: e_i = expected net revenue per acre for agricultural enterprise i;

σ_i = standard deviation of the per acre net revenue from agricultural enterprise i;

$\sigma ii = \sigma_i^2$, i.e., variance of the per acre net revenue from agricultural enterprise i;

σ_{ij} = covariance of the per acre net revenue from agricultural enterprises i and j;

q_i = acres of the total acreage allocated to agricultural enterprise i.

In addition, the above relationships are subject to:

$$q_i \geq 0 \text{ and } \sum_{i=1}^{n} q_i \leq Z$$

where: Z = total acres available.

Other constraints may be placed on q_i as needed to meet crop rotation or other economic or agronomic restrictions.

To compute the points in each E,V frontier by hand would require considerable effort. Therefore, a quadratic programming algorithm is used to solve for the E,V frontiers in this analysis. By parametrically varying the level of income, points along the E,V frontier are generated. These points outline or denote the position of the frontier. The quadratic program provides an expected net revenue and corresponding variance (standard deviation) for each parametric solution. Also, the underlying crop mix or cattle-cash crop mix (i.e. the "activities" or q_i of the quadratic program solution) giving rise to that level of income and variance are provided by the program.

The E,V solution procedure can be cast as a quadratic programming problem of the following general form:

$$\text{Maximize:} \quad F(q) = q'I - q'Bq$$

$$\text{subject to:} \quad q_1 + q_2 + q_3 + q_4 + q_5 + q_6 + q_7 + q_8 \leq 1$$

$$q'I = K$$

$$q_i \geq 0$$

where: q = a 1 x 8 row vector of enterprise proportions for the seven crops and one cattle alternative[6/];

I = a 1 x 8 column vector of expected income (e_i) for the seven crops and one cattle alternative;

[6/] The cattle alternative was varied between alternative 1, 2, 3, 4, 5, or 6, depending on which alternative was to be used in the estimation of the cattle E,V frontier.

B = a 8 x 8 variance-covariance matrix for the seven crops and one cattle alternative;

K = a constant that varies parametrically from 0 to a maximum possible value.

The objective function represents the expected income (q'I) minus the variance of that income (q'Bq) subject to the total land restriction (1) and other restrictions on the individual enterprises. The total land restriction (Z) was set equal to one acre, so that when solved the individual enterprises would be a fraction of an acre. The results could then be extended to any size of operation by multiplying by total acreage. The parametric solution obtains that maximum difference between expected income and variances for each level of expected income (q'I). This is the same as finding the minimum variance for each level of expected income, i.e. the E,V frontier (Halter and Dean).

Using the data on incomes, variance and covariance generated in Section II, E,V frontiers were estimated for each crop and cattle alternative. Specifically, the q_i or activities presented in the above equations are:

q_1 = alfalfa

q_2 = corn for grain

q_3 = corn for silage

q_4 = dry beans

q_5 = malting barley

q_6 = feed barley

q_7 = sugar beets

q_8 = cattle feeding alternative (which varied between alternatives 1, 2, 3, 4, 5 and 6).

The first E,V frontiers estimated were for the seven cash crops, without restrictions on the crops. (The cattle feeding alternative, q_8, was removed from the program by setting q_i = 0). The next E,V frontiers estimated were for the seven cash crops with agronomic restrictions on minimum and maximum percentage of specific crops, i.e. $q_1 \geq .20$, $q_3 \leq .25$, $q_4 \leq .50$, $q_2 + q_6 \leq .30$, $q_5 + q_7 \leq .40$ and $q_8 = 0$. E,V frontiers were then estimated for each of the six cattle alternatives developed in Section II, resulting in a total of 24 E,V frontiers. The cattle frontiers were initially constrained to require that cattle feeding account for at least 60 percent of the acreage, i.e. $q_8 \geq .60$, with all crop constraints removed. This restriction was later relaxed to .20 and finally allowed to assume the most efficient level to arrive at the "optimal" proportion of cattle feeding. The expected net revenues (e_8) and associated variance (σ_{88}) and covariance terms (σ_{8j}) were adjusted for each of the six cattle alternatives to reflect the underlying sets of gain and feed ration assumptions. The complete set of E,V frontiers are presented in Woolery. An abbreviated set of results, representing the more significant relationships of the analysis, are below.

Estimated E,V Frontiers

The frontiers presented in this paper are measured in terms of net income and variance (standard deviation)[7/] for the entire 575 acres of the hypothetical farm. These aggregate values are arrived at by multiplying the per acre values derived in Woolery by the 575 acres of cropland assigned to the representative farm. The crop mix giving rise to specific points on the frontier are also converted to total acres basis by expressing

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[7/] The quadratic programming solution procedure is specified in terms of variance and covariance. However, for ease of presentation, the results (concerning variability) are expressed as standard deviations.

as a proportion of the 575 acres rather than as a percent of one acre. The tables accompanying the frontiers provide detailed information on crop mixes, corresponding incomes and standard deviations as well as confidence intervals about that expected income.

The following frontiers are an abridged subset of the total set of frontiers estimated by Woolery. Those presented here are deemed to be most "representative" of the crops and systems available to the typical producer in the Big Horn Basin. These frontiers, particularly with respect to the cattle feeding alternatives, are generally superior to those not discussed in that their relative position indicates universally higher income and/or lower risk (variance) when compared with other combinations.

Specifically, the results presented here are for the cash crop systems and two of the cattle systems (numbers 1 and 5). Within each, some special cases are examined. These include the constrained and unconstrained cases for cash crops, with different income time periods being analyzed (1956-76 and 1973-76) for both cash crops and integrated systems.

Crop Systems

Within the Big Horn Basin the individual producer has a fairly wide range of commodities from which to choose. In addition to such cash crops as sugar beets, malting barley and dry beans, producers may also elect to grow lower valued crops such as alfalfa, corn and feed barley. Each combination of the above crops will give rise to different levels of total income as well as different levels of variance or risk for that income. The E,V frontiers provide such information.

The frontiers discussed in this section refer to four specific crop frontiers. These include a frontier representing expected income and variance to be derived from various combinations of crops using the income data for the entire time period (1956-76) with no agronomic restrictions on minimum or maximum acreage. The results of this estimation are presented in Table 9. A second frontier uses the same time series but is constrained by minimum and maximum averages for specific crops (Table 10).

The third frontier represents the expected income-variance relationships for the same crops but uses the more recent or subjective income measure (1973-76). The variance also is derived from detrended data to arrive at a random measure of variance. The fourth frontier is the same as three with the exception that agronomic constraints are again imposed upon the crop mix. The results of these last two frontiers are presented in Table 11 and 12, respectively.

An initial observation concerning the results in the tables is that the land area (i.e. the crop mix proportions) was initially divided between most of the crops. As the parametric solutions proceeded and incomes increased, crops were dropped from the solution until at the maximum only one crop was left in each solution. Such an adjustment (of "dediversifying" or specializing) is needed to realize the higher income levels. Correspondingly, however, the level of risk associated with the higher incomes increased at a more rapid rate than income.

Another feature of these tables is the general importance of malt barley in the crop mixes and the failure of sugar beets to assume a major position in the crop mixes in the longer time period analysis. Sugar beets did become important, however, at the maximum income levels in the more recent time period. Also, dry beans do not appear to be a good crop in

Table 9. Expected Income-Variance (E,V) Trade-offs and Corresponding Crop Mixes for the 1956-76 Period Using Total Variance, Unconstrained Cash Crop Case.

Expected Gross Margin (Income)	Standard Deviation (Risk)	Crop Mix (Acres)							95% Confidence Interval		68% Confidence Interval	
		Alfalfa	Corn for Grain	Corn for Silage	Dry Beans	Malting Barley	Feed Barley	Sugar Beets	Lower Bound	Upper Bound	Lower Bound	Upper Bound
$ 47,484*1	$26,128	150.7	67.3	123.6	-	205.9	-	-	$- 4,773	$ 99,740	$21,356	$ 73,612
55,718	30,694	113.9	69.6	137.4	-	253.0	-	1.2	- 5,670	117,105	25,024	86,412
59,645	32,913	84.5	68.4	142.0	-	277.2	-	2.9	- 6,181	125,471	26,726	92,558
63,572	35,150	55.2	67.3	146.6	-	301.3	-	4.6	- 6,728	133,872	28,422	98,722
67,499	37,410	25.9	66.1	151.8	-	324.9	-	6.3	- 7,320	142,318	30,090	104,909
71,375*2	39,646	-	63.3	154.7	-	349.0	-	8.1	- 7,918	150,667	31,729	111,021
74,854	41,676	-	48.3	143.8	-	372.6	-	10.4	- 8,499	158,206	33,178	116,530
78,332	43,729	-	33.9	132.8	-	395.6	-	12.1	- 9,125	165,784	34,604	122,061
81,811	45,793	-	19.6	121.9	-	419.2	-	14.4	- 9,775	173,397	36,018	127,604
85,290	47,875	-	5.2	111.0	-	442.2	-	16.1	-10,459	181,039	37,415	133,165
102,694*3	58,616	-	-	-	-	575.0	-	-	-14,536	219,926	44,080	161,311

*For expositional purposes, enumerated asterisks refer to specific points on Figure 1. Each point on Figure 1 thus represents the income,variance and crop mix as presented in the table.

terms of minimizing the overall level of risk for given income levels, in view of the low proportion of dry beans in any combination.

Two of the frontiers (Tables 9 and 11) do not consider any agronomic restrictions on crop mix. Given that most producers do follow some rotational patterns, an analysis of efficient crop mixes in the presence of some typical restrictions is perhaps useful. These results are presented in Tables 10 and 12.

As evidenced by Table 10, the constraint on malting barley and sugar beets results in dry beans replacing the constrained crops in the maximum solution. The crop mix between respective expected incomes (i.e. 1956-76 vs. 1973-76) are about the same.

The information presented in Tables 9 through 12 shows that changes in crop mix and proportions will affect the level of income and commensurate risk or variance. Each specific combination on a particular table is "optimal" or "best" in that no other combination of crops can provide that level of income at that corresponding level of risk. Thus, if a producer desires a given level of income at some risk level, the table will provide the crop mix which will yield that income, given the assumptions and constraints of the model.

The incomes are drawn from an assumed normal distribution, and while these incomes are the expectation or mean income, some fluctuations around that mean will occur. Using the variance measure (standard deviation), confidence intervals can be constructed for each income. The 95 and 68 percent confidence intervals for each income (and hence crop mix giving rise to that income) are provided in each table. By using the lower bound estimates, producers may be confident that a specific crop mix will yield an income greater than the lower bound income at least 97.5 or 84 percent of the time. The degree of confidence that a producer is willing

Table 10. Expected Income-Variance (E,V) Trade-offs and Corresponding Crop Mixes for the 1956-76 Period Using Total Variance, Constrained Cash Crop Case.[a/]

Expected Gross Margin (Income)	Standard Deviation (Risk)	Crop Mix (acres)							95% Confidence Interval		68% Confidence Interval	
		Alfalfa	Corn for Grain	Corn for Silage	Dry Beans	Malting Barley	Feed Barley	Sugar Beets	Lower Bound	Upper Bound	Lower Bound	Upper Bound
$47,484*4	$26,128	150.7	67.3	123.6	-	205.9	-	-	$- 4,773	$ 99,740	$21,355	$73,612
51,802	28,509	143.2	70.7	132.3	-	228.3	-	-	- 5,215	108,819	23,293	80,310
52,308	28,790	119.6	82.8	142.6	-	230.0	-	-	- 5,273	109,888	23,518	81,098
52,446	28,894	115.0	93.2	134.6	2.9	230.0	-	-	- 5,342	110,233	23,552	81,340
52,785*5	29,199	115.0	87.4	125.9	16.1	230.0	-	-	- 5,612	111,182	23,587	81,984
55,016	32,332	115.0	51.2	70.7	108.1	230.0	-	-	-9,649	119,681	22,684	87,348
57,310	36,817	115.0	-	-	191.5	230.0	38.5	-	-16,324	130,945	20,493	94,128
57,759	38,019	115.0	-	-	217.4	230.0	12.7	-	-18,279	133,797	19,740	95,778
57,977*6	38,715	115.0	-	-	230.0	230.0	-	-	-20,027	135,407	19,263	96,692

[a/] As noted in the text, the results generated in this table required that specific minimum acreages be allocated to crops to meet generally accepted agronomic recommendations; i.e., minimum of 115 acres in alfalfa and a maximum of 230 acres in dry beans, malt barley and sugar beets.

[*] For expositional purposes, enumerated asterisks refer to specific points on Figure 1. Each point on Figure 1 thus represents the income,variance and crop mix as presented in the table.

Table 11. Expected Income-Variance (E,V) Trade-offs and Corresponding Crop Mixes Using Random Variance and 1973-76 Period Income, Unconstrained Cash Crop Case.

Expected Gross Margin (Income)	Standard Deviation (Risk)	Crop Mix (Acres)							95% Confidence Interval		68% Confidence Interval	
		Alfalfa	Corn for Grain	Corn for Silage	Dry Beans	Malting Barley	Feed Barley	Sugar Beets	Lower Bound	Upper Bound	Lower Bound	Upper Bound
$ 75,728*1	$ 4,893	149.5	26.5	147.8	-	79.9	169.6	2.3	$ 65,941	$ 85,514	$ 70,834	$ 80,621
96,393	6,877	-	29.9	201.8	-	132.3	201.8	8.6	82,639	110,147	89,516	103,270
109,607	8,372	-	19.0	216.2	1.7	178.3	144.9	15.0	92,863	126,351	101,235	117,979
129,766	10,925	-	-	236.9	10.9	247.3	57.5	23.0	107,916	154,491	118,841	140,691
147,484	13,334	-	-	221.4	16.7	307.1	-	29.9	120,917	174,254	134,251	160,919
157,378	14,737	-	-	184.0	18.4	338.7	-	33.4	127,903	186,852	142,640	172,115
167,176	16,192	-	-	146.6	20.7	370.9	-	36.8	134,792	199,560	150,984	183,368
181,867	18,452	-	-	90.9	23.6	418.0	-	42.6	144,963	218,770	163,415	200,319
191,665*2	19,987	-	-	53.5	25.9	449.7	-	46.0	151,691	231,639	171,678	211,652
205,747	22,235	-	-	-	27.6	495.7	-	51.8	161,276	245,721	183,511	227,982
207,587	22,540	-	-	-	16.1	503.1	-	55.2	162,507	252,667	185,047	230,127
210,174	23,012	-	-	-	-	514.6	-	60.4	164,151	256,197	187,163	233,186
211,761	24,593	-	-	-	-	437.6	-	137.4	162,576	260,947	187,157	236,354
212,934	24,760	-	-	-	-	380.7	-	194.3	163,415	260,453	188,175	237,694
215,878	37,070	-	-	-	-	238.6	-	336.4	141,738	290,019	178,808	252,948
220,823*3	58,248	-	-	-	-	-	-	575.0	104,328	337,318	162,576	279,071

*For expositional purposes, enumerated asterisks refer to specific points on Figure 2. Each point on Figure 2 thus represents the income,variance and crop mix as presented in the table.

Table 12. Expected Income-Variance (E,V) Trade-offs and Corresponding Crop Mixes Using Random Variance and 1973-76 Period Income, Constrained Cash Crop Case.[a]

Expected Gross Margin (Income)	Standard Deviation (Risk)	Crop Mix (Acres)							95% Confidence Interval		68% Confidence Interval	
		Alfalfa	Corn for Grain	Corn for Silage	Dry Beans	Malting Barley	Feed Barley	Sugar Beets	Lower Bound	Upper Bound	Lower Bound	Upper Bound
$ 74,859[*4]	$ 4,824	176.0	22.4	143.8	-	80.5	150.1	2.3	$ 65,211	$ 84,508	$ 70,035	$ 79,684
91,868	6,544	118.5	33.9	143.8	-	131.7	138.6	8.6	78,781	104,955	85,324	98,411
103,282	7,929	115.0	29.9	143.8	2.3	170.2	100.1	13.8	87,423	119,140	95,352	111,211
116,248	9,626	115.0	16.7	143.8	13.8	212.2	55.8	17.8	96,997	135,499	106,622	125,873
117,846	9,902	115.0	-	143.8	29.3	213.9	56.9	16.1	98,043	137,649	107,945	127,748
118,743	10,080	115.0	-	143.8	38.5	215.1	47.7	15.0	98,584	138,903	108,664	128,823
119,773	10,310	115.0	-	137.4	48.3	215.6	44.9	14.4	98,003	150,392	109,463	130,082
121,521	10,753	115.0	-	116.2	62.7	217.4	51.2	12.7	100,016	143,026	110,768	132,273
122,688	11,075	115.0	-	102.4	72.5	218.5	55.2	11.5	100,539	144,837	111,613	133,762
124,436[*5]	11,598	115.0	-	81.7	86.8	220.2	62.1	9.8	101,240	147,631	112,838	136,034
127,351	12,552	115.0	-	46.6	111.0	223.1	72.5	6.9	102,247	152,456	114,799	139,903
132,745	14,542	115.0	-	-	158.1	228.3	71.9	1.7	103,661	161,828	118,203	147,286
137,149	16,382	115.0	-	-	203.0	230.0	27.0	-	104,386	169,913	120,767	153,531
139,955	17,681	115.0	-	-	230.0	223.1	-	6.9	104,593	175,318	122,274	157,636
140,547	18,389	115.0	-	-	230.0	194.9	-	35.7	103,770	177,324	122,159	158,936
141,134	19,498	115.0	-	-	230.0	166.2	-	63.8	102,137	180,130	121,636	160,632
142,313	22,707	115.0	-	-	230.0	109.3	-	120.8	96,899	187,726	119,606	165,019
144,572[*6]	31,090	115.0	-	-	230.0	-	-	230.0	82,392	206,753	113,482	175,663

[a] As noted in the text, the results generated in this table required that specific minimum acreages be allocated to crops to meet generally accepted agronomic recommendations: i.e., minimum of 115 acres in alfalfa and a maximum of 230 acres in dry beans, malt barley and sugar beets.

[*] For expositional purposes, enumerated asterisks refer to specific points on Figure 2. Each point on Figure 2 thus represents the income, variance and crop mix as presented in the table.

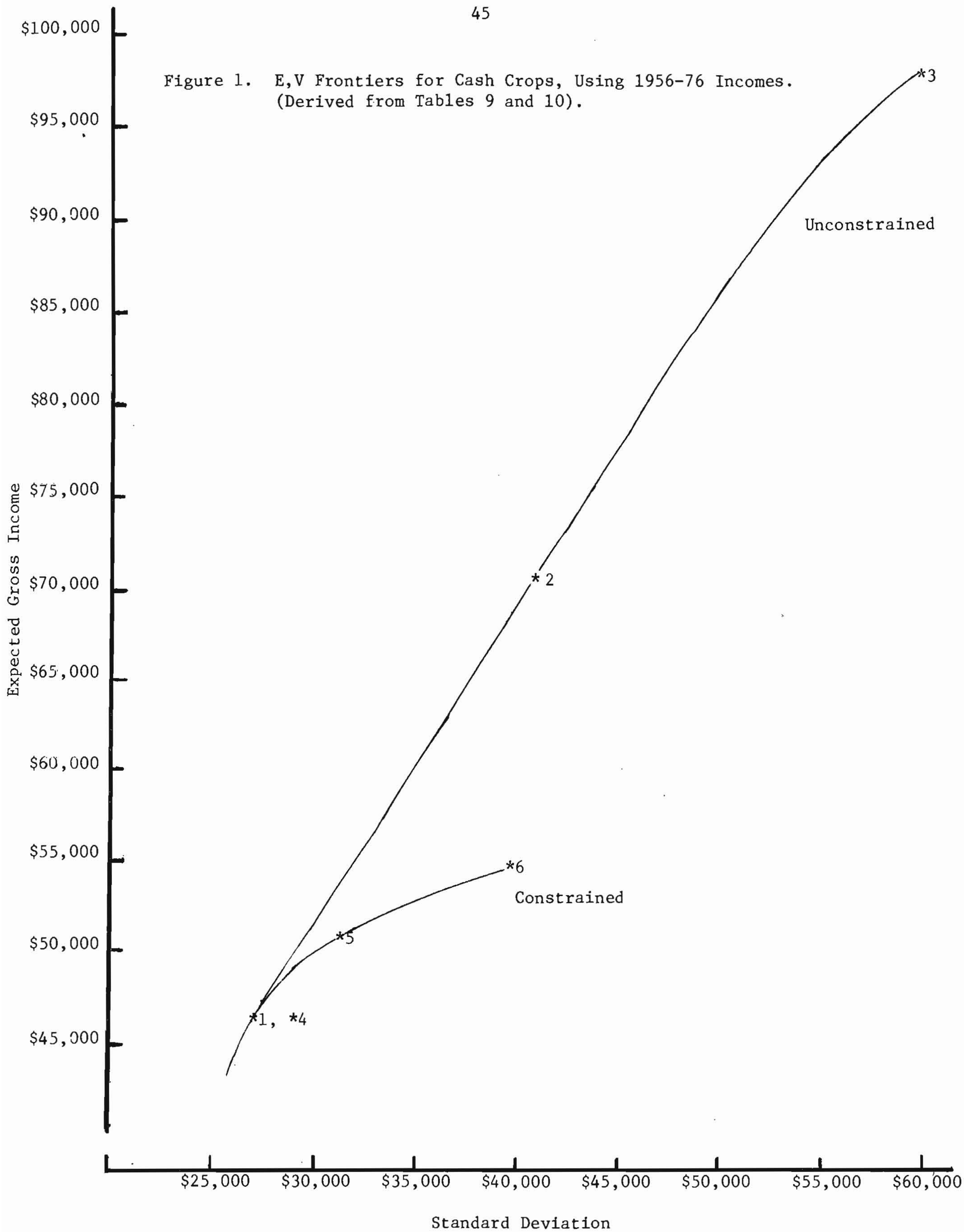
to accept is, of course, a matter of personal preference. It should also be noted that the confidence intervals displayed in Tables 8 and 9 are not strictly valid statistical measures. This is because the standard deviation in these cases is from detrended data (thus not the "true" or total standard deviation) and the income mean is for an abridged time period, (1973-76) not the entire 21 year period. However, within a given table, the relative magnitudes of the confidence interval remain valid.

The graphical counterpart of the tabular information is presented in Figures 1 and 2. Specific incomes and variances from the tables and the underlying crop mixes are denoted on the figures by enumerated asterisks. These figures depict the E,V frontiers for the incomes, variance (standard deviations) and crop mixes presented in the tables (each dot or point on the frontier represents the crop mix giving rise to that income and variance as taken from the corresponding table). Specifically, Figure 1 presents the E,V curves for the objective income (1956-1976) constrained cases. Figure 2 presents the relevant curves for the subjective income (1973-76) cases.

The relative positioning of the frontiers is important, in that one curve is deemed superior to another if it lies to the left of the latter curve. This is due to the concave nature of the curves. That is, the more leftward a curve lies, the lower the standard deviation (risk) at any given income level. Alternatively, the higher or upper curve represents higher income levels at a given level of risk. Also, the general shape of the curves (concave) is due to variance increasing more rapidly than income at very high income levels.

As noted on the figures, the unconstrained cases are superior to the constrained. This verification of the Le Chatelier Principle is not

Figure 1. E,V Frontiers for Cash Crops, Using 1956-76 Incomes. (Derived from Tables 9 and 10).



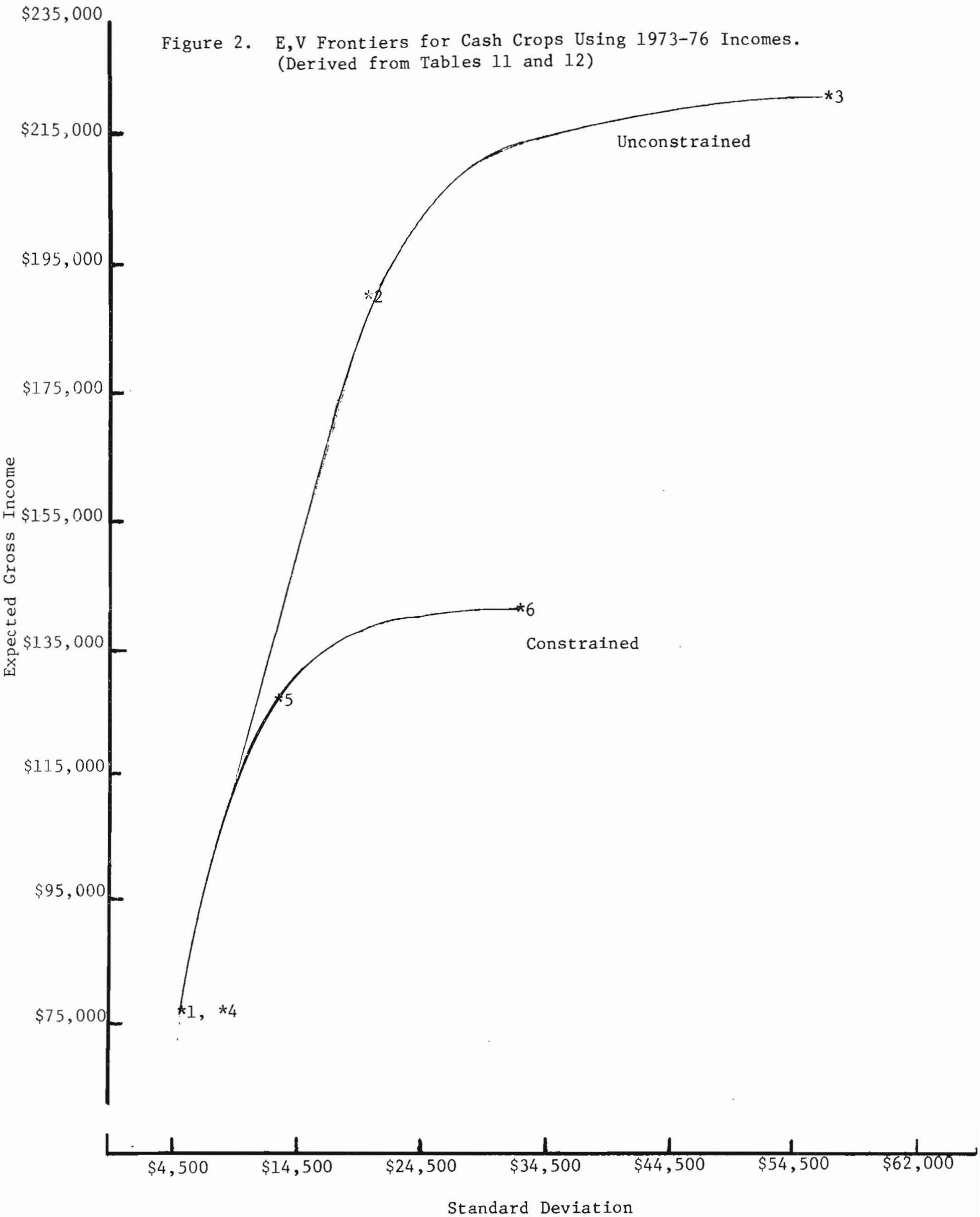


Figure 2. E,V Frontiers for Cash Crops Using 1973-76 Incomes. (Derived from Tables 11 and 12)

surprising given the underlying mathematical programming solution. From an empirical standpoint, the lower incomes in the constrained case results from fewer alternative crop mixes, particularly those needed to realize the very high income levels. While on different graphs, it can also be observed that the subjective frontiers are superior to the objective. This observation again is not surprising, given that the subjective incomes are much greater due to the high crop prices of the 1973-76 period as compared with the entire period from 1956-1976.

The important relationship between the two types of frontiers (objective vs. subjective) is any observed differences in the underlying crop mixes which give rise to those incomes. There are some significant differences in crop mix as observed by comparing Table 9 and 10 with 11 and 12. What this suggests is that producers' expectations concerning future prices will be instrumental in their choice of the most representative frontier and "optimal" crop mix. If producers give greater weight to more recent years, they will deem the subjective frontier to be most representative. The choice of optimal crop mix should then be from the corresponding frontier. It is usually suggested that producers do give greater weight to more recent observations. If this is true, then the implications are that producers in the Big Horn Basin would favor the production of cash crop systems such as malting barley and sugar beets over integrated or more diverse systems. Recent production data from the area would support this supposition.

The implication of producers giving greater weight to the more recent years is that it may overlook the cyclical nature of certain commodity prices, such as those for cattle. The following section discusses the relative merits of integrating cattle feeding with cash crops. Given the high cattle prices observed in 1978 and 1979, some combination of cattle and crops may indeed be attractive.

Integrated Cash Crop-Cattle Feeding Systems

Among the eight producers interviewed at the outset of this study, only two currently allocate a portion of crop production to the feeding of cattle. Several producers acknowledge a history of cattle feeding but had recently shifted from a mixed crop-cattle system to one of crops exclusively. Such a shift does not appear to be unique to this set of producers but rather has occurred across the general population of growers in the region. Reasons for the shift may be the relatively high crop prices observed over the last five growing seasons (1973 to 1978) as well as a personal preference for working conditions associated with crops vis a vis livestock. The continued persistence of integrated systems within the region as well as the change in relative output prices in 1979 suggests that such integrated systems merit inclusion in any analysis of the region.

As noted above, only two of the six cattle feeding systems will be discussed here (1 and 5). The assumptions underlying these alternatives are presented in Section II. Within these two alternatives, the same income and variance specifications as used in the cash crops will be assessed i.e. objective income (1956-76), subjective income (173-76) and total random variance. In addition, the allocation of acreage between cash crops and forage (initially set at 40% and 60% respectively) will be relaxed to arrive at the "optimal" or most efficient allocation of the acreage between cattle and cash crops. The results of E,V trade-offs can be compared with the underlying crop mixes and frontier shapes and location associated with the crop systems.

Cattle alternative 1 is derived from a situation featuring 400 pound calves fed at a 2.0 pound per day rate of gain. Of the 60 percent of the acreage devoted to feed crops, the mix under this alternative features corn silage, feed barley and alfalfa hay. The remaining 40 percent of the acreage is available for the production of cash crops. As with the cash crop analysis, both objective and subjective income cases are assumed. The objective case again covers the 21 year period whereas the subjective income case covers the 1973-76 period. The only constraint on cash crops is the 40 percent maximum acreage on cash crops in total.

The results of the cattle alternative 1 income-variance relationship are presented in Table 13 for the objective case. The results for the subjective case are presented in Table 14. As was the case for crops, the expected income increases with the more recent period.

Cattle alternative 5 is similar to alternative 1 except that it eliminates feed barley from the forage crop mix (leaving corn silage, alfalfa hay and beet tops). The 40 percent of acreage devoted to cash crops may assume any mix of cash crops. Both objective and subjective income cases are again examined. The results for this analysis are presented in Tables 15 and 16, respectively.

In examining the results (incomes, standard deviations and crop mixes) across these two alternatives, comparisons should be made between results using the same income period. Thus, the results in Table 13 for alternative 1 feature the same assumptions as Table 15 for alternative 5. Similarly, Table 14 results may be compared with Table 16.

The patterns of cash crop production observed across each alternative are quite similar, with the objective cases being dominated by malt barley

Table 13. Expected Income-Variance (E,V) Trade-offs for Cattle Alternative 1 Using Incomes and Variance for the 1956-76 Period.

Expected Gross Margin (Income)	Standard Deviation (Risk)	Crop Mix (Acres)								95% Confidence Interval		68% Confidence Interval	
		Cattle Feeding	Alfalfa	Corn for Grain	Corn for Silage	Dry Beans	Malting Barley	Feed Barley	Sugar Beets	Lower Bound	Upper Bound	Lower Bound	Upper Bound
$44,752	$24,564	345.0	33.4	35.7	27.6	9.2	78.8	-	8.1	$-4,376	$ 93,880	$20,188	$ 69,316
47,622	25,536	345.0	43.1	40.8	35.1	7.5	90.9	-	8.1	-3,450	98,693	22,086	73,157
50,842	26,703	345.0	23.6	40.8	39.7	6.3	110.4	-	9.8	-2,565	104,248	24,139	77,545
52,417	27,307	345.0	11.5	40.8	42.0	5.2	120.2	-	10.4	-2,197	107,031	25,110	79,724
54,688	28,215	345.0	-	38.0	41.4	4.6	134.6	-	11.5	-1,742	111,119	26,473	82,904
58,184	29,687	345.0	-	24.7	31.1	2.3	158.1	-	13.8	-1,190	117,559	28,497	87,872
61,680	31,251	345.0	-	11.5	20.1	0.6	181.7	-	16.1	- 822	124,183	30,435	92,932
65,130	32,879	345.0	-	-	7.5	-	204.7	-	18.4	- 627	130,887	32,252	98,009
66,252	33,419	345.0	-	-	-	-	213.3	-	16.7	- 587	133,090	32,833	99,671
67,062	33,931	345.0	-	-	-	-	230.0	-	-	- 799	134,924	33,132	100,993

Table 14. Expected Income-Variance (E,V) Trade-offs for Cattle Alternative 1 Using Incomes for the 1973-76 Period and Random Variance.

Expected Gross Margin (Income)	Standard Deviation (Risk)	Crop Mix (Acres)								95% Confidence Interval		68% Confidence Interval	
		Cattle Feeding	Alfalfa	Corn for Grain	Corn for Silage	Dry Beans	Malting Barley	Feed Barley	Sugar Beets	Lower Bound	Upper Bound	Lower Bound	Upper Bound
$ 65,636	$15,951	345.0	-	46.6	105.2	56.9	-	-	21.3	$33,735	$ 97,537	$49,686	$ 81,587
66,798	15,956	345.0	-	43.1	102.9	59.8	-	-	24.2	34,885	98,710	50,842	82,754
69,374	16,002	345.0	-	35.7	98.3	65.6	-	-	31.1	37,369	101,378	53,372	85,376
71,950	16,094	345.0	-	27.6	93.2	71.3	-	-	38.0	39,761	104,138	55,856	88,044
82,599	16,658	345.0	-	-	77.6	75.9	34.5	-	42.0	49,283	115,914	65,941	99,257
97,296	17,704	345.0	-	-	21.3	79.4	81.7	-	47.2	61,887	132,704	79,592	115,000
104,627	18,343	345.0	-	-	-	69.6	107.5	-	52.9	67,942	141,312	86,285	122,970
106,467	18,538	345.0	-	-	-	58.1	115.0	-	56.4	69,391	143,543	87,929	125,005
108,307	18,768	345.0	-	-	-	46.6	123.1	-	60.4	70,771	145,843	89,539	127,075
111,067	19,153	345.0	-	-	-	29.3	135.1	-	65.6	72,761	149,374	91,914	130,220
112,907	19,441	345.0	-	-	-	17.8	142.6	-	69.6	74,026	151,789	93,466	132,348
115,667	19,918	345.0	-	-	-	0.6	154.7	-	74.8	75,831	155,503	95,749	135,585
116,156	20,119	345.0	-	-	-	-	133.4	-	96.6	75,917	156,394	96,037	136,275
116,742	20,746	345.0	-	-	-	-	105.2	-	124.8	75,250	158,234	95,996	137,488
118,922	25,915	345.0	-	-	-	-	-	-	230.0	67,091	170,752	93,006	144,837

Table 15. Expected Income-Variance (E,V) Trade-offs for Cattle Alternative 5 Using Incomes and Variance for the 1956-76 Period.

Expected Gross Margin (Income)	Standard Deviation (Risk)	Crop Mix (Acres) Cattle Feeding	Alfalfa	Corn for Grain	Corn for Silage	Dry Beans	Malting Barley	Feed Barley	Sugar Beets	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound	68% Confidence Interval Lower Bound	68% Confidence Interval Upper Bound
$48,335	$26,784	345.0	35.1	30.5	10.9	16.1	75.3	-	11.5	$-5,233	$101,902	$21,551	$ 75,118
51,204	27,675	345.0	44.9	35.7	18.4	14.4	87.4	-	11.5	-4,146	106,553	23,529	78,879
54,614	28,819	345.0	35.1	38.0	25.3	13.2	105.8	-	12.7	-3,025	112,252	25,795	83,433
56,189	29,377	345.0	23.6	38.0	27.0	12.1	115.6	-	13.2	-2,565	114,943	26,812	85,566
58,558	30,257	345.0	5.8	38.0	29.9	11.5	130.0	-	14.4	-1,955	119,071	28,302	88,815
62,140	31,671	345.0	-	27.0	22.4	9.2	154.1	-	16.7	-1,202	125,482	30,469	93,811
65,636	33,143	345.0	-	13.8	12.1	7.5	177.7	-	19.0	- 650	131,922	32,493	98,779
69,190	34,724	345.0	-	-	-	5.8	201.3	1.2	21.3	- 259	138,638	34,466	103,914
70,110	35,150	345.0	-	-	-	-	209.3	-	20.7	- 190	140,409	34,960	105,260
70,415	35,311	345.0	-	-	-	-	215.6	-	14.4	- 207	141,036	35,104	105,725
71,128	35,782	345.0	-	-	-	-	230.0	-	-	- 437	142,692	35,345	106,920

Table 16. Expected Income-Variance (E,V) Trade-offs for Cattle Alternative 5 Using Incomes and Variances for the 1973-76 Period.

Expected Gross Margin (Income)	Standard Deviation (Risk)	Crop Mix (Acres)								95% Confidence Interval		68% Confidence Interval	
		Cattle Feeding	Alfalfa	Corn for Grain	Corn for Silage	Dry Beans	Malting Barley	Feed Barley	Sugar Beets	Lower Bound	Upper Bound	Lower Bound	Upper Bound
$ 72,778	$18,216	345.0	-	33.9	93.7	73.6	-	-	28.2	$36,346	$109,210	$54,562	$ 90,994
76,515	18,262	345.0	-	23.0	86.8	82.2	-	-	38.0	39,991	113,039	58,253	94,777
79,097	18,343	345.0	-	15.0	81.7	88.0	-	-	44.9	42,412	115,782	60,755	97,440
87,067	18,722	345.0	-	-	64.4	92.6	23.0	-	50.0	49,623	124,511	68,345	105,789
101,758	19,659	345.0	-	-	8.1	96.0	70.7	-	55.2	62,439	141,076	82,099	121,417
106,272	20,022	345.0	-	-	-	81.7	88.0	-	61.0	66,229	146,315	86,250	126,293
108,112	20,206	345.0	-	-	-	69.6	95.5	-	64.4	67,701	148,523	87,906	128,317
109,957	20,413	345.0	-	-	-	58.1	103.5	-	68.4	69,132	150,782	89,545	130,370
112,717	20,769	345.0	-	-	-	40.8	115.6	-	73.6	71,179	154,255	91,948	133,486
114,557	21,039	345.0	-	-	-	29.3	123.0	-	77.6	72,479	156,636	93,518	135,597
117,317	21,471	345.0	-	-	-	12.1	135.1	-	82.8	74,376	160,258	95,847	138,788
119,232	21,804	345.0	-	-	-	-	142.6	-	87.4	75,624	162,840	97,428	141,036
119,640	21,982	345.0	-	-	-	-	122.5	-	107.5	75,676	163,605	97,658	141,623
120,233	22,552	345.0	-	-	-	-	94.3	-	135.7	75,130	165,336	97,681	142,784
122,182	26,715	345.0	-	-	-	-	-	-	230.0	68,753	175,611	95,467	148,896

Figure 3. E,V Frontiers for Cattle Alternative 1 and 5, 1956-76 Periods for Incomes and Variance.

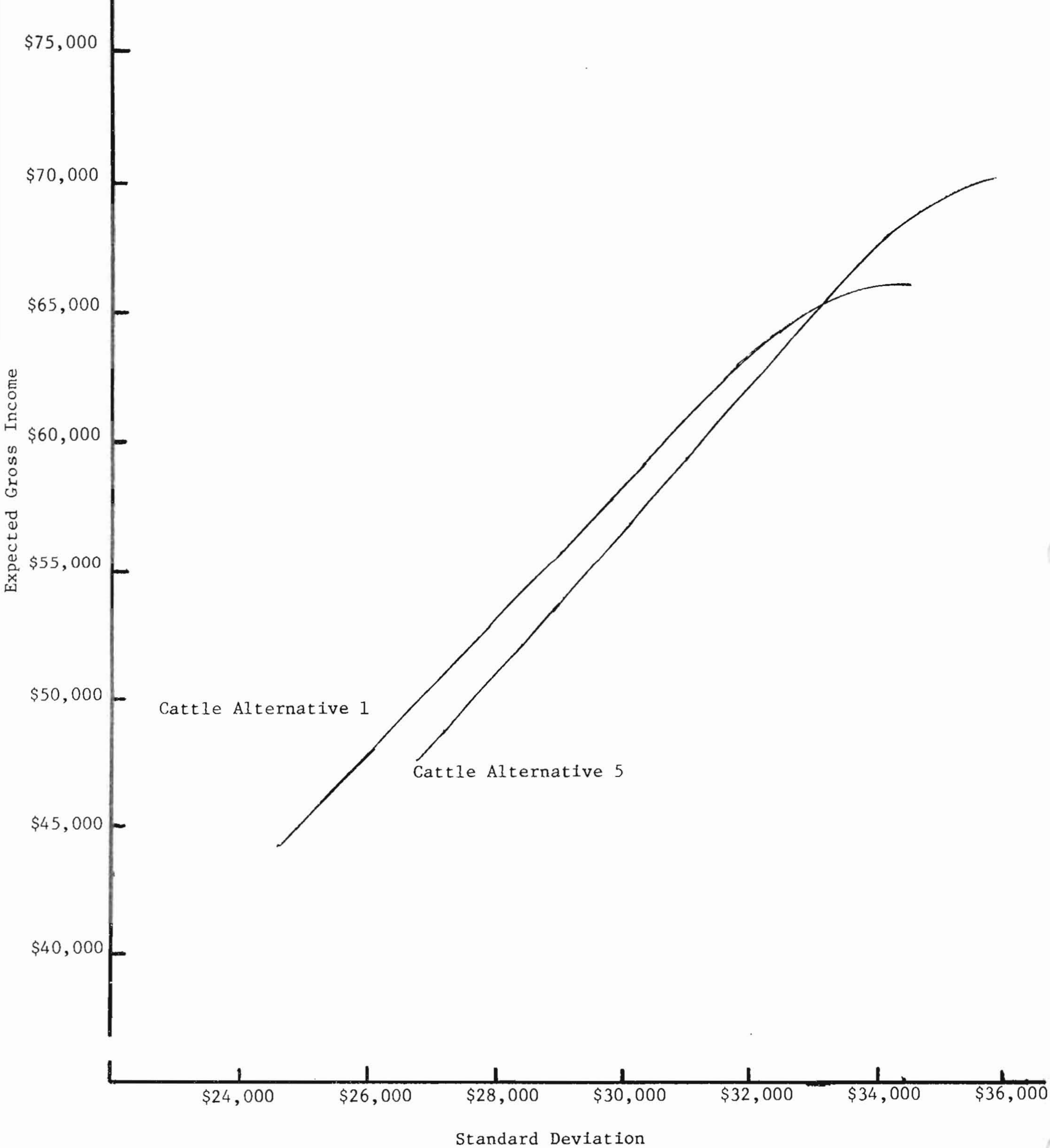
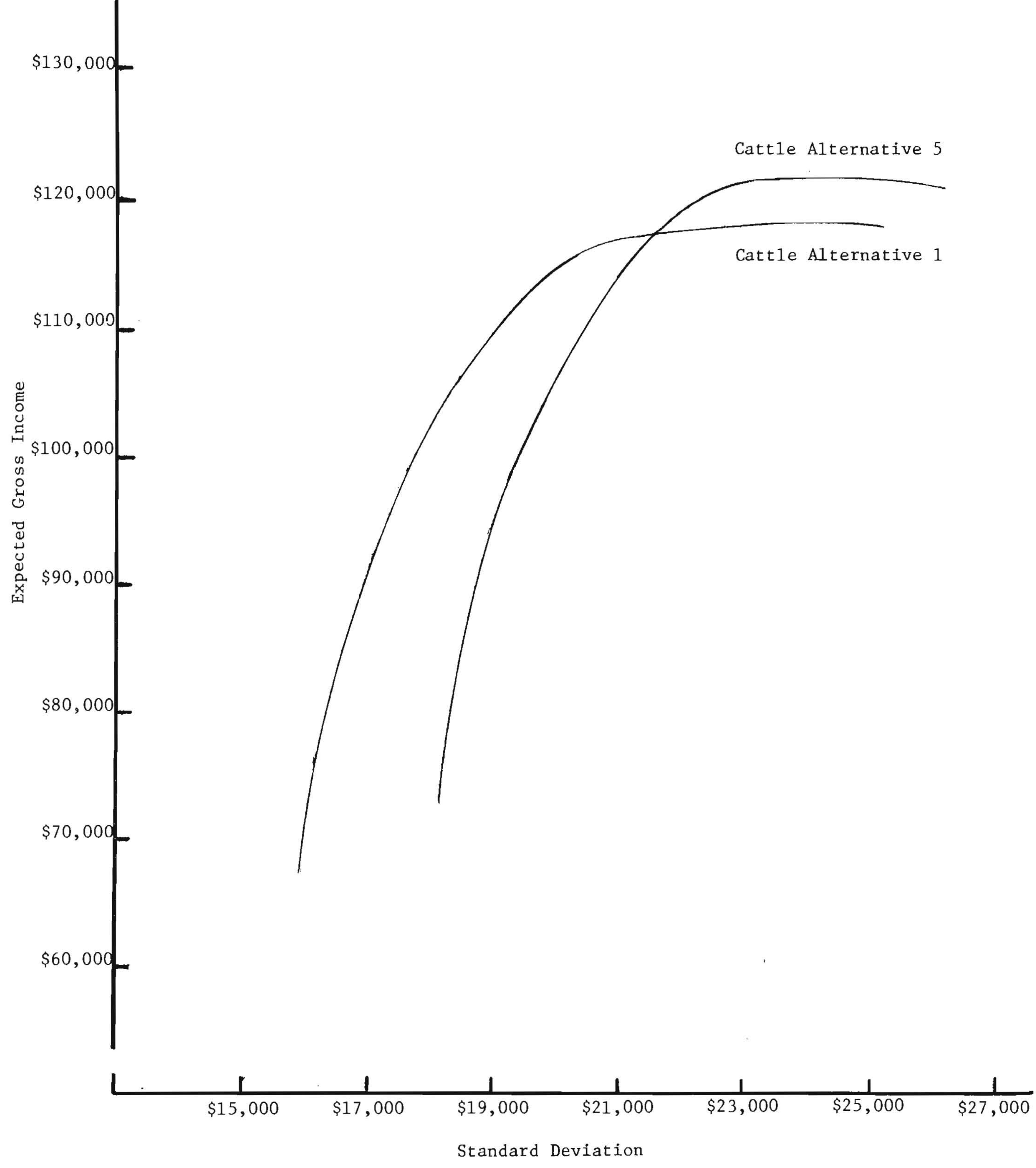


Figure 4. E,V Frontiers for Cattle Alternatives 1 and 5, 1973-76 Income and Random Variance.



whereas the subjective cases feature sugar beets at the high income levels. (This observation is consistent with the crop mixes observed for the cash crop alternatives under objective and subjective cases).

One notable feature is the slightly higher incomes recorded under alternative 5 as compared with alternative 1 for both income periods. Given that the only differences in assumptions are the differences in forage mixes, this difference in expected income is due to the slightly higher total carrying capacity under the forage crop mix in alternative 5.

The relative positioning of the resultant E,V frontiers can serve to indicate superiority of alternative systems. As a point of comparison, the frontiers for Tables 13 and 15 are graphed in Figure 3 (objective cases). Those for Tables 14 and 16 (subjective) are presented in Figure 4.

As noted by the positioning of the curves on each figure, cattle alternative 1 is generally superior (same income at lower level of risk) except at very high income levels. The curves intersect with alternative 5 becoming superior to alternative 1. This intersection arises from the effect of the covariance term on the quadratic solution procedure. That is, there is a relatively high covariance between sugar beets and alternative 1. As a result, when sugar beets increased in the cash crop mix under alternative 1, the variance tended to rise much more rapidly than income, causing the frontier to level off.

A final analysis of the trade-off between risk and income under alternative crop and cattle feeding systems concerns the "optimal" mix of cattle feeding for the given land base. What proportions, if any, of the land base would be assumed by cattle feeding if the 60 percent restriction were relaxed? To address this question, the 60 percent proportion of forage acreage initially assigned to cattle feeding was allowed to vary and hence find some optimal or efficient proportion. (The 60-40 ration was suggested by producers during the interview procedure).

The case analyzed here is cattle alternative 1 under the objective income situation. The results of the analysis are presented in Table 17. By examining the changing proportions of cropland devoted to forage preduction (the column headed "cattle feeding") the proportion of the acreage devoted to cattle rises to a maximum percentage of 47 and then declines to 20 percent at the maximum income. For a producer with a moderate degree of risk aversion, the implication of these results is that a mix of from about 40 to 50 percent of crop acreage in cattle feeding crops may be "optimal". Again, however, any point along a frontier is efficient. The specific mix selected by the individual producer would depend upon his attitude toward risk.

A Comparison of Cattle and Cash Crop Alternatives

Tables 9 through 12 presented income and risk relationships for cash crops. Tables 13 through 17 deal with integrated systems of cash crops and cattle feeding. In terms of providing information to producers one needed to compare across these different systems to assess the relative advantages of one system vis a vis alternatives.

A comparison of cattle alternatives with cash crop alternatives revealed several points. First, using incomes for the entire 21 year period placed cattle feeding superior to cash crops, whereas the reverse was true when only the more recent income period was used to formulate expected income. These later results are due to a negative relationship between cattle prices and crop prices over the period of subjective

Table 17. Optimal Cattle Feeding Activity as a Proportion of Total Crop Acreage: Cattle Alternative 1 Using the 1956-76 Period.[a]

Expected Gross Margin (Income)	Standard Deviation (Risk)	Crop Mix (Acres)								95% Confidence Interval		68% Confidence Interval	
		Cattle Feeding	Alfalfa	Corn for Grain	Corn for Silage	Dry Beans	Malting Barley	Feed Barley	Sugar Beets	Lower Bound	Upper Bound	Lower Bound	Upper Bound
$47,788	$23,443	185.2[b]	95.5	56.4	77.1	-	145.5	-	4.6	$ 903	$ 94,674	$24,346	$ 71,231
		(32.2)	(16.6)	(9.8)	(13.4)	-	(25.3)	-	(.8)				
52,678	25,864	203.6	62.1	54.6	79.9	-	167.9	-	6.9	949	104,403	26,812	78,539
		(35.4)	(10.8)	(9.5)	(13.9)	-	(29.2)	-	(1.2)				
57,368	28,233	221.4	20.7	51.8	81.1	-	190.9	-	9.2	903	113,833	29,135	85,600
		(38.5)	(3.6)	(9.0)	(14.1)	-	(33.2)	-	(1.6)				
65,521	32,465	249.0	-	21.3	54.1	-	235.8	-	15.0	592	130,450	33,057	97,986
		(43.3)	-	(3.7)	(9.4)	-	(41.0)	-	(2.6)				
72,973	36,495	270.8	-	-	-	-	279.5	-	19.0	17	145,964	36,478	109,469
		(47.1)	-	-	-	-	(48.6)	-	(3.3)				
76,090	38,278	249.6	-	-	-	-	308.8	-	16.7	-466	152,645	37,812	114,368
		(43.4)	-	-	-	-	(53.7)	-	(2.9)				
81,564	41,814	199.5	-	-	-	-	364.6	-	10.9	-2,064	165,192	39,750	123,378
		(34.7)	-	-	-	-	(63.4)	-	(1.9)				
90,816	48,691	115.0	-	-	-	-	-	-	460.0	-6,567	188,198	42,067	139,507
		(20.0)	-	-	-	-	-	-	(80.0)				

[a] As noted in the text, cattle feeding is allowed to assume any proportion of the total acreage, rather than the 60 percent constraint imposed in other analyses.

[b] Values in parentheses represent percentages of total acreage.

income, i.e. 1973 through 1976. It has been noted that in the Big Horn Basin area over the period 1973 to 1976, producers were moving from integrated feeding operations to strictly cash crops, perhaps in response to high relative crop prices.

Also under the objective incomes the change from random to total variance caused a reversal in the results (i.e. cash crops were superior to cattle feeding at lower income levels under the random case). One would attribute this outcome to the relatively greater random variance (as a percent of total variance) found in the cattle alternatives when compared to cash crops. Under the subjective income the change from total to random variance did not reverse the results of the E,V frontiers, as the corps were already superior, but it did increase the distance between the cash crops and cattle feeding frontiers. This clearly shows the effect the random variance has on the solution to the E,V frontiers. The question remains, "Does random or total variance better represent the producers' views on risk?" Similarly, "Does the objective or subjective income more accurately represent the producers' income expectiations?"

In terms of empirical content, the information developed in the E,V analysis provides useful information to producers concerning the selection of farm plans. Specifically, producers may use underlying confidence intervals to establish the income level an enterprise mix would provide at selected probabilities.

The producer could also compare his present crop mix with the mixes provided by the E,V analysis. This can be accomplished by taking a weighted average of the producers' crops and obtaining the approximate expected gross margin and standard deviation as provided by his mix.

This then may be compared with a corresponding income level on the frontier, representing the most efficient mix of enterprises at that income level. Such a comparison would allow the producer to explicitly compare the trade-off between risk and income under his current system with that in a risk-minimizing crop mix for the same income level.

In a more general sense, the E,V analysis provides the producer with an explicit look at the trade-off between return and risk, which may be of value to the younger producer (given the lack of sufficient observations to establish subjective probabilities concerning income). The E,V analysis may also provide the more experienced producer a better understanding of the effects of diversification on risk and return than that provided by experience.

SUMMARY

Agricultural producers typically adjust their decision processes in response to changing economic conditions. Many decisions, such as crop mixes producers choose to undertake, are based upon expectations concerning final prices and yields, and the actual revenues realized from a given action will usually differ from expectations due to imperfect knowledge. The greater the potential for divergence (between expectations and realization) the more uncertain or risky is that particular course of action, assuming constant risk aversion on the part of producers.

Agricultural researchers continue to devote considerable effort to understanding both the effects of risk on producer decision-making and to provide information to producers to assist in decision-making. While rather sophisticated methodologies have been suggested for inclusion of risk in agriculture modeling, a gap appears to exist between these elegant models and their applicability in real world decision-making.

This paper has discussed the results of an application of one of the more basic risk frameworks to agricultural decision making in the Big Horn Basin. Specifically, income and risk measures are presented for individual crop and cattle feeding alternatives, based upon historical data. These results were then extended into the more complete analysis of a hypothetical 575 acre farm enterprise via E,V or portfolio analysis to suggest some efficient combinations of crops and cattle alternatives.

Within the E,V framework, alternative specifications of the income and variance parameters were assessed for both cropping systems and

integrated crop-cattle feeding systems. To the extent that such alternative specifications affect the efficient combinations of crops, the applicability of the results to producers in the Basin must be tempered by producers' expectations. Further, agronomic constraints were also introduced into the analysis to reflect current cultural practices.

The results indicate the nature of the relationship involved in the trade-off between income and risk, i.e. the higher the expected income the greater the risk. The results also indicate the effects of diversification (or conversely, specialization). Single crop mixes may display high relative incomes but also high variance or variation of income. The constrained cases result in lower income (and risk), as does the use of the lengthier time period of analysis (1956-76). The E,V frontier represents an efficient combination of crops (yielding a specific income under the assumptions of the analysis), and no point on the frontier is "better" than another; each point represents a combination of crops providing a specific income at a minimum level of risk.

The inclusion of cattle feeding as an alternative enterprise resulted in higher income and/or lower risk for the model formulation featuring income over the 21-year period. The negative correlation between cattle prices and cash crops over this period tends to make such combinations "stabilizing" in terms of income. Using the more recent time period (1973-76), integrated systems have lower risk but also much lower income due to high crop prices relative to those observed for cattle. The relative output price structure in 1978-79, however, would lend support to the use of lengthier time periods. Within cattle feeding alternatives, the efficient mix appears to be as high as 47 percent of the land area devoted to cattle forage production. This is somewhat less than the 60 percent allocation cited by those producers who currently engage in integrated cattle-cash crops enterprises.

The results of this analysis suggest that producers expectations are weighted by more recent observations concerning output prices. Using the more recent time period (1973-76) to derive the income parameters, E,V frontiers for cash crops systems feature a predominance of malt barley and sugar beets in the resultant crop mix with extremely high attendant incomes. The emphasis on these crops in current cropping systems indicates that producers expect similar price relationships to continue. However, the strong upward movement in cattle prices in the late 1970's will perhaps enhance the appeal of more integrated systems, as existed in the preceeding two decades.

The importance of sugar beets in the efficient cropping systems portrayed by the E,V frontier estimation raises a question concerning the effects of a loss of sugar beet contracts within the Basin. Such a loss of contracts occurred recently in the Pacific Northwest due to closure of Utah and Idaho sugar processing facilities. The removal of sugar beets as an option in the crop mix would place greater emphasis on dry beans and malt barley. Given the high variability of income associated with dry beans, the overall level of risk would increase. Producers would be forced to choose a less efficient (less profitable) crop mix. Should the price premium now being afforded malt barley in relationship to feed barley be narrowed, the situation between risk and income (with respect to the trade-off) would worsen. Compensating increases in cattle incomes could offset such losses, making integrated systems a viable option.

The selection of an optimum crop mix undertaken and the resultant amount of risk borne is a matter of individual producer choice. Personal preferences and attitudes towards risk will impinge upon the final selection of a crop mix. This study provides information to be used in performing a comparative assessment of a wide range of cropping and integrated systems, including explicit measures of the trade-off between risk and income. With such a comparison, producers may be able to judge the relative efficiency of current systems and hence evaluate the potential gains realized from shifting to alternative systems.

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APPENDICES

Supporting Data

Appendix Table A-1. Net Revenues for Seven Big Horn Basin Cash Crops, 1956-1976, per Acre.[a]

Year	Alfalfa	Corn for Grain	Dry Beans	Corn for Silage	Malting Barley	Sugar Beets	Feed Barley
	---------------------- Dollars per Acre ----------------------						
1956	12.43	3.74	16.34	-7.69	89.62	114.98	9.04
1957	5.13	2.77	21.27	-10.20	114.80	49.93	18.55
1958	4.09	14.62	8.92	8.52	118.33	58.62	38.09
1959	7.44	21.81	17.17	27.56	167.76	49.74	16.30
1960	5.92	14.62	22.16	17.08	98.50	44.42	15.67
1961	9.27	13.03	24.79	12.56	110.00	18.85	22.44
1962	12.57	15.33	1.41	14.57	113.74	39.64	20.28
1963	19.91	12.07	12.01	15.81	112.13	68.90	19.94
1964	16.11	24.20	17.48	23.81	136.08	28.56	29.90
1965	20.05	10.45	33.26	11.75	147.35	7.81	33.89
1966	14.62	29.38	14.76	31.38	123.41	72.06	28.25
1967	25.10	22.67	43.26	29.83	149.86	95.06	33.95
1968	14.01	32.24	13.77	37.13	146.01	90.26	28.97
1969	15.75	34.99	36.09	40.85	161.22	95.05	36.38
1970	20.04	30.60	26.88	28.76	160.35	82.27	33.06
1971	22.10	37.69	69.17	42.39	130.32	109.05	42.07
1972	24.70	57.00	61.49	50.59	217.80	151.49	45.14
1973	52.22	146.54	349.32	107.40	267.16	422.24	89.92
1974	69.82	104.38	320.44	61.07	429.17	701.93	125.87
1975	70.33	94.96	135.28	77.94	446.95	233.04	125.94
1976	75.83	77.53	39.02	133.71	310.47	178.95	106.34

[a] Gross Margin (revenue) above variable costs.

Source: Yahya and Adams; Wyoming Agricultural Statistics, 1978.

Appendix Table A-2. Net Revenues[a/] for Selected Cattle Feeding Alternatives, Big Horn Basin, 1956-1976.

Year	Cattle Alternative 1	2	3	4	5	6
	Dollars per Acre					
1956	15.22	6.19	17.27	7.90	16.94	6.55
1957	105.49	47.43	110.48	50.34	116.08	50.67
1958	49.78	26.61	52.27	28.43	59.06	31.48
1959	49.22	28.82	51.58	30.63	61.72	36.91
1960	35.30	18.35	37.15	16.84	43.94	23.52
1961	38.00	20.63	39.85	22.06	45.75	25.02
1962	17.81	-1.58	19.07	-.84	20.29	-2.43
1963	2.80	-13.05	4.34	-12.13	3.01	-15.66
1964	39.52	26.17	41.34	27.65	45.32	29.83
1965	77.67	62.51	79.60	64.90	84.48	68.14
1966	25.44	9.78	27.56	10.60	46.42	24.72
1967	57.94	40.08	60.83	42.49	69.92	48.64
1968	121.23	100.92	124.96	104.72	144.92	108.79
1969	138.42	113.69	140.38	114.85	163.26	134.01
1970	98.48	73.59	102.09	76.61	112.66	84.01
1971	150.33	121.40	155.80	125.98	161.96	128.50
1972	204.18	167.60	211.09	173.37	240.92	197.46
1973	-9.78	-59.14	-8.86	-59.44	-9.61	-69.13
1974	77.56	59.88	81.66	63.34	71.26	65.94
1975	177.93	154.25	184.60	160.45	202.04	174.95
1976	108.97	78.87	114.12	83.14	128.78	93.36

[a/] Gross margin above variable costs.

Source: Agee, 1969, 1971, 1973, 1974; Yahya and Adams; Kearl, 1968, 1974; USDA, 1976 Agricultural Statistics.

Appendix Table A-3. Net Revenues[a] for Selected Cropping Programs[b], Big Horn Basin, 1956-1976.

Year	Feed Crop Programs A	Feed Crop Programs B	Cash Crop Programs I	Cash Crop Programs II	Cash Crop & Feed Programs Combined I A	Cash Crop & Feed Programs Combined II A	Cash Crop & Feed Programs Combined I B
	Dollars per Acre						
1956	1.97	-1.28	107.90	114.98	44.34	47.17	42.40
1957	2.22	-5.31	67.70	49.93	28.41	21.30	23.89
1958	16.90	7.11	74.98	58.62	40.13	33.59	34.26
1959	19.63	21.15	82.07	49.74	44.61	31.67	45.52
1960	14.21	13.52	59.23	44.42	32.22	26.29	31.81
1961	14.97	11.51	43.82	18.85	26.51	16.52	24.44
1962	15.94	13.93	59.94	39.64	33.54	25.42	32.35
1963	18.01	17.12	80.74	68.90	43.10	38.36	42.57
1964	24.06	21.35	58.01	28.56	37.64	25.86	36.02
1965	20.55	14.40	46.03	7.81	30.74	15.45	27.05
1966	26.75	26.04	86.13	72.06	50.50	44.87	50.07
1967	30.11	28.32	110.07	95.06	62.09	56.09	61.02
1968	29.53	29.76	105.53	90.26	59.93	53.82	60.07
1969	33.98	32.85	113.17	95.05	65.66	58.41	64.98
1970	28.22	25.98	103.66	82.27	58.40	49.84	57.05
1971	37.88	35.92	114.88	109.05	68.68	66.35	67.50
1972	43.24	42.34	116.65	151.49	93.81	86.54	93.26
1973	89.88	89.81	379.76	422.24	205.83	222.82	205.79
1974	83.45	63.86	627.22	701.93	300.96	330.84	289.21
1975	91.45	75.51	291.63	233.04	171.52	148.09	161.96
1976	112.48	115.26	214.98	178.95	153.48	139.07	155.14

[a] Gross margin above variable costs.

[b] See Section II for a discussion of each cropping system.

Appendix Table A-4. Net Revenues[a/] for Integrated Cash Crops and Cattle Feeding Alternatives[b/], Big Horn Basin, 1956-1976.

Year	Cattle Alternative					
	1	2	3	4	5	6
	Dollars per Acre					
1956	52.29	46.88	56.35	50.73	53.32	47.09
1957	90.37	55.54	86.26	50.18	96.73	57.48
1958	59.86	45.95	54.81	40.51	65.42	48.88
1959	62.36	50.12	50.85	38.28	69.86	54.97
1960	44.87	34.70	40.06	27.87	50.06	37.80
1961	40.33	29.90	31.45	20.77	44.98	32.54
1962	34.66	23.03	27.30	15.35	36.15	22.51
1963	33.17	24.47	30.17	20.28	34.10	22.90
1964	46.91	38.91	36.23	28.01	40.50	41.10
1965	64.49	55.92	50.88	42.06	69.10	59.30
1966	49.71	40.32	45.36	35.18	62.30	49.28
1967	78.79	68.08	74.52	63.52	85.98	73.21
1968	114.95	102.76	111.08	98.93	129.17	107.48
1969	128.32	113.48	122.25	106.93	143.23	125.68
1970	100.55	85.62	94.16	78.87	109.06	91.87
1971	136.15	118.79	137.10	119.21	143.13	123.05
1972	190.37	168.42	187.25	164.62	212.41	186.34
1973	146.04	116.52	163.58	133.23	146.14	110.42
1974	297.42	286.81	329.77	318.78	293.65	290.45
1975	223.41	209.20	203.97	189.49	237.88	221.62
1976	151.37	133.31	140.05	121.46	163.26	142.01

[a/] Gross margin above variable costs.

[b/] See Section II for a discussion of each alternative.